Statistical independence in quantum mechanics

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Statistical independence when applied to spaces and not just variables become a powerful tool for encapsulating quantum objects like fields and particles. It has been used on particle descriptions to generate base properties. It also can be used on massless fields. The Borexino solar neutrino survival data correlates well with an analysis for the electron neutrino, ν_e being massless. The ν_e remains a massless charge free field as proposed by Pauli. The ν_e wave function generates a spatial density oscillation that reduces its scattering potential by one-half. The analysis produces a confirmation of the original quantum conjecture by Planck and Einstein that radiation is quantized and can be explicitly verified from the allowed solutions to the field state equations of their statistically independent embedded space. This data exposes one of the major defects in the foundation of quantum mechanics, the missing mechanism that quantizes a massless field and generates inertia for particles. *draft August 1, 2016*

CONTENTS

I. INTRODUCTION

Unfortunately the foundation of quantum mechanics suffers from a number of defects that have never been corrected. Two principal defects are the missing mechanism that quantizes the massless fields and generation of inertia and self-energy of massive particles. These questions have not been answered by either quantum electrodynamics or the high energy effective field theories, because such models fail to strictly conserve energy by using embedded singularities or Lagrangian methods (Wallace and Wallace, 2016). Starting with high energy scattering data and working backwards to understand the structure of particles has some major disadvantages. In contrast experimenting at very low energies has two advantages: it will reduce the collateral confusion of particle creation and increase the scale that can be more easily accessed in the laboratory. The statistical nature of quantum measurement is a singular characteristic requiring consideration of when a statistical basis is required for such work and how this requirement is actually generated? Therefore this paper presents a method for understanding a physical statistically independent space.

A. Statistical Questions

Physical statistical independence is a quantum concept and not connected to classical Brownian motion, the central limit theorem, or the many particle processes of statistical mechanics. Mark Kac continued a very productive line of research by studying diffusion and Brownian motion (Kac, 1947) (Kac, 1949), and in the late 1940s his work was subsequently used as a quantum description of non-relativistic path integral analysis, which looked much like diffusion. Kac's work is important because of the need to understand the quantum diffusion of hydrogen or its isotopes in compact metals. This is a cross over application that can be treated as a combination of classical diffusion with quantum potential wells.

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In high density metals where the proton is treated as unscreened in shallow potential well whose bound eigenvalues are determined by the mass of the isotopes (Wallace, 2011). The minimum potential well depth supporting an eigen state yields a very curious result. This minimum bound state is for a boson with a mass relationship equivalent to the Compton relation that is independent of the potential wells depth and only dependent on its radius (Wallace and Wallace, 2011).

In the middle of the 1960s, Kac posed another interesting problem (Kac, 1966): Can one hear the shape of a drum with holes? In the 1950s Kac's mathematical interests were applied to discrete random processes and the concept of statistical independence of random variables (Kac, 1959). Kac realized that part of the statistical nature of dynamics in quantum mechanics involves the less than well defined nature of the quantum particle. He thought the symbolism of a drum head might strike his physics colleagues as something familiar. There is a basic question that underlies both subjects and that is the precise definition of information; types of information and its origin. This very classical problem of a drum head's spectral analysis was an efficient way of extending Dirac's model of incorporating longitudinal fields into a relativistic quantum model (Dirac, 1932), because it provided a physical platform for those waves. Once a spectral content emerged the problem of information defined by a physical entity became a problem that could be solved.

The random walk generated by Brownian motion was a nice analogy but not physically realistic for fleshing out the statistical basis of quantum mechanics. The path integral approach that Kac was exploring with Feynman was an attack on the dynamics problem of quantum objects; unfortunately, their elementary particles entered in as Newtonian point masses and point charges. The fields entered in as plane or spherical waves. These assumptions remove consideration of the particle structure. Kac's consideration of a defective drum made the particle a real structure to be considered. The scale of the drum head removes the point particle assumption as a barrier to having a particle with a defined scale. The complex spectra of the drum was a start at producing a statistical basis for a scattering particle's properties and immediately brought into question the information content of the spectra.

Kac, with his drum analogy, was able to take the problem to a lower level and in a different direction by showing that the inversion techniques fail to determine the true structure of the drum and requires more data than can ever be collected from the resulting spectral response. Kac's inversion analysis is similar to the potential problem, that of determining an unknown potential's structure from a scattering experiment such as done with accelerators. Kac's analogy works on two levels, first the drum is defined in a physically exact space with no inherent uncertainties, and two, as a realistic quantum object the drum has to be a fuzzy object with fuzzy holes. Mathematically exact structures come with a high cost requiring the use of the continuum. Whereas a collection of precise properties require less of demand on the physical space. Secondly the analysis is taken from the point of view of an external viewer doing the measurement. This is simply the particle scattering problem restated. The ability to unambiguously invert the scattering data to generate the structural description is fruitless because the particle generates a set of basics properties for measurement: mass, charge, spin, and magnetic moment. So the question should be how and why does this replacement in outcomes occur in the physical world?

B. Extending the Field Analysis

Instead of the acoustical problem that Kac posed, the problem was recast into one of a near-field electromagnetic scattering problem using multiple simultaneous frequencies to examine an unknown object's: range, scale, conductivity and magnetic permeability (Siegfried, 1983). Maxwell's equations can be solved explicitly for the forward problem, and checked against known materials (Dodd and Deeds, 1968). Then unknown materials can be used and their conductivity and permeability properties can be accurately extracted if they behave with the following two material restrictions.

$$\begin{aligned} \mathbf{B} &= \mu \mathbf{H} \\ \mathbf{J} &= \sigma \mathbf{E} \end{aligned} \tag{1}$$

From the solutions of the forward problem the domain of allowed solutions can be plotted out and the allowed reflection responses are defined by the restrictions found in Eq. 1 (Wallace, 2011). As long as these conditions are in place a rough inverse problem can be solved to the precision of the measurements and produce useful information. The analysis failed rather spectacularly if the material is capable of absorbing the incoming energy and processing it into an excited quantum state on the scale of the object being examined. The object then becomes Kac's drum head.

The Faraday-Maxwell description now needs to be extended, but there is not much in the scattering data to tell one how to accomplish this task, except that the resultant dynamic magnetic field appears to be a longitudinal field that deeply penetrates a magnetic conductor, which normally would not allow such a penetration. In particular an experiment must be done to measure the dispersion relations of the newly observed fields; therfore, new data is acquired to resolve mass (Wallace, 2009a) (Wallace, 2009b). The dispersion relation for a well annealed iron or a low carbon steel produces three results but the most interesting is a propagating fields with a mass of 10^{-9} of an electrons mass. This very light spin wave has a correspondingly large scale $\sim .14$ meters. A low frequency magnetic field drives the creation of exciton that is an oscillating longitudinal wave with mass and large scale structure. This solves a major experimental problem because now a quantum particle structure can be examined on a lab bench. It turns out the structure of this particle at relativistic energies matches the behavior of a spin zero boson and not a fermion. This light boson can collect in large numbers because of its low mass a Bose-Einstein condensate can form at temperatures that exceed the Curie point of the metal making it even easier to study.

The only way to salvage the analysis is to test how primitive waves, if they are perturbed, both transverse and longitudinal will then behave. What has to be resolved is a modified wave to explain the mass of this magnetic exciton. This light exciton must be treated from the beginning as a relativistic particle because of the very small mass. This was done for the longitudinal spin wave (Wallace and Wallace, 2014). However, there was still work to be done because the Faraday-Maxwell version of electromagnetism is not quantized. Quantizaton is an assumed not a generated addition to the properties of a simple field. Complicating this are two possible primitive field types: transverse and longitudinal. To describe the spin wave starting from the massless field dispersion relation needs a randomizing process to localized the field producing inertia. This gives a structural expression for the field that is transformed into a particle representation with an inertial mass. It also uncovered another important aspect of randomization process. The randomized primitive longitudinal spin wave established a space in (r, τ) coordinates in three spatial dimensions and time that is a statistically independent of the laboratory frame (Wallace and Wallace, 2014) (Wallace and Wallace, 2015). In this independent space a scale for the particle is generated and hence its mass defining the particle self-energy.

In contrast a transverse field that is a propagating front needs a different randomization process affecting the phase of the field front. It is this process that now can be verified with some recent neutrino survival data. Its not inertial mass that is expected to be produced from transverse field randomizing rather the quantizing of its energy. In both cases the quantized self-energy of a field $\hbar\omega$ and of a particle's self-energy mc^2, are the result. Here the analysis is restricted to the three-dimensional embedded statistically independent spaces.

The mathematical feature that appeared when randomizing the longitudinal field was a second order differential equations that had two solutions. The solutions corresponded to a boson and a fermion each having an inertial mass. This is prior to the fields acquiring a spin. The structual analysis was further verified when the charge characteristics of the two particle types were derived, showing a mass independent charge quantization for the fermion and a weak charge with more complex symmetry properties for the boson. The angular momentum properties are secondary properties and are generated in the laboratory frame for each family of particles (Wallace and Wallace, 2016), as are all other vector and tensor properties of matter. The symmetry of the entities density functions in the statistically independent self-reference frame satisfies the *U(1)* symmetry group of spherical symmetry and only produces scalar properties. To bring in the two high energy families of particles requires an analysis that includes both two and one dimensional statistically independent embedded spaces. As these lower dimensional components cannot participate individually in radiative processes allowing their energy densities to be greater.

C. Statistically Independent Spaces

Introducing the concept of a statistically independent space compatible with quantum mechanics and relativity was done in 2014 to describe the relativistic longitudinal spin wave as simply a necessary requirement. It is a simple concept that extends the idea of statistically independent random variables to a space. Normally to define a new space (r, θ, ϕ) a few axioms are laid down and properties of elements in that space are then associated with operations like addition and multiplications. Then the transformation rules are introduced of how the new space maps into other spaces. Here those mapping rules are member of the null set when a particle's statistically independent space is mapped to the laboratory frame where measurements are made.

There are relations between the two frames, but not coordinate transformations. There is a basic problem the laboratory frame, (r', θ', ϕ'), where measurements are typically done is a physical space where there is a minimum comparison scale of the dimension of the smallest stable particle and that scale is not sharp but fuzzy. A comparison measurement or a set of measurements then defines a position or a length that comes with a well defined error band. Since, the new space, (r, θ, ϕ), is reference to a specific particle or field at its current center of symmetry determined by a random process makes the structure impossible to oriented. There is no information contained about the angular coordinates in this new space $(r, \theta, \phi) \Rightarrow (r)$. The transformation

$$r \Rightarrow r' \quad r(r', \theta', \phi') \;\; does \;\; not \;\; exist. \tag{2}$$

The angular variable θ and ϕ are not measures that have any meaning externally in the laboratory frame even though the self-reference space has the same number of physical dimensions. The only feature that is left for measurement in the laboratory frame is ϵ, the mean random displacement. It is not the coordinate transformations that are of interest, but the actually generation and storage of information that the independent space permits, so it is not a total loss and actually essential in constructing measurable information about particles and fields. It is only from this basic property information about particles and fields that abstract information that makes up this page can be constructed. This new particle or object based space has been named the self-reference frame and two examples follow.

II. QUANTUM FIELD

It is difficult to make the case for the electron neutrino, ν_e, having mass unless the definition of mass is changed. The apparent missing solar ν_e from better experiments have continued to find a deficit of neutrinos even at lower energies (Derbin and group, 2016). This data now allows a test to be made against proposed neutrino structures. The neutrino representation that we are familiar with started with a question asked by a freshman in a physics class in 1966: What are the implications of a complex mass in the solutions of the Schrödinger equation? The instructor did not know and said he would ask a theorist. A few months later G. Feinberg publish a paper with a particle having a complex mass by using the Klein-Gordon equation to generate plane wave solutions for the faster than light tachyon (Feinberg, 1967). It was unfortunate the Klein-Gordon equation was used as the energy operator is incorrectly applied to Eq. 3 in its derivation and as a consequence the Klein-Gordon equation violates the conservation of energy relation (Wallace and Wallace, 2014). This defect (see appendix B) is the sources of the problems with the time dependence of the probability densities that result from its solution (Srendnicki, 2007). The proposed tachyon has yet to be discovered.

$$E^2 = c^2p^2 + (mc^2)^2 \tag{3}$$

There was an additional problem the original question of a complex mass exposed. The only experimental definition of mass that exists with a structural feature comes from the Compton experiment. Where mass is found to be structurally related to a scale parameter call the Compton wavelength for which we will use the symbol, ϵ (Heisenberg, 1930) (Miller, 1994).

$$m = \frac{\hbar}{\epsilon c} \tag{4}$$

It is possible to derive this mass to scale relationship in other ways (Wallace and Wallace, 2011), but there is a simple physical approach to dealing with inertial mass being generated from a field. A real mass represents an entity that can be localized, therefore taking something more primitive such as a longitudinal field and asking how can a field be interfered with to acquire inertia and be localized is a way to generate mass. To treat the concept of mass in general arising out the localization of a primitive field, two cases must be considered, the transverse and the longitudinal fields. Assume these fields can have an opposite component an anti-field that it can annihilate. The pair of fields required for this random process bring into question virtual fields and particles allowed by the time-energy version of Heisenberg uncertainty principle. The virtual entities are allowed in pairs, which introduce no new information such as charge, angular momentum, or linear momentum. These three items must sum to be zero over the virtual pair. If this were not the case new information would be introduced and that could only be with a real cost in energy. This implies that a single virtual photon would violate this restriction because of its angular momentum.

The confusion about field behavior goes back to the middle of the 19th century and James C. Maxwell when he considers the substrate upon with the electromagnetic properties of space are generated (Maxwell, 1866). He followed Torricelli thoughts from the 18th century that the only interacting components of matter were mass and charge points. This assumption was unwarrented. To justify static potential interactions he felt a substrate was necessary for the interactions. We have found the electrostatic potential is simply a reflection of the particle sturcture (Wallace and Wallace, 2015). The next problem left in this area is the concept that the photon is not a primitive field, but is constructed from a more primitive basis. A quantized field is a condition usually applied as an assumption and not derived from physical process.

A. Inertia

What is required to generate a mass from a primitive field are obstacles to aid in localizing a field moving at the speed of light. A set of obstacles that conserve energy in the laboratory frame are composed of field-anti-field pairs. Sometime the original field makes it through and other times it annihilates and its opposite number takes over being the propagating field. This results in a random displacement. If this process is truly random then the original field will be localized under some very specific conditions. Our original field's a self energy is taken as $\hbar\omega$ as well as for our final field as energy is conserved. To compute the rate of pair production the self-energy of the new pair becomes $2\hbar\omega$ with a mass equivalent equal to $2mc^2$. The localization is initiated in the laboratory frame so that the rate, R, of the pair-production can be computed from the Heisenberg uncertainty relation for energy.

$$R = \frac{1}{\delta t} \leq \frac{4mc^2}{\hbar} \tag{5}$$

At any time our field has a 50% chance of encountering a pair and compounding that a 50% chance of annihilating and passing the baton to the newly minted field. So in total it has a 25% chance of being replaced. This rate turns into an equality since the only virtual field pairs that can interact with original field must have the identical energy as these are conservative processes. This rate of replacement is 1/4 the rate of pair production.

$$\frac{R}{4} = \frac{mc^2}{\hbar} \tag{6}$$

The inverses of the rate *R/4* is a mean interval any particular field lives and the distance light can travel in that interval is ϵ which now can be computed from Eq. 6.

$$\epsilon = \frac{\hbar}{mc} \tag{7}$$

This is the Compton relation produced from a real disorder paramter, ϵ. The net effect on our field is set by the mean rate of exchanging fields and generating a locality for a particle with inertia as its local position is unknown to a mean random value ϵ. The angular coordinate description is lost in the self-reference frame as it is reset to the present position of the particle's center of symmetry. By randomizing the local location of the fields center of symmetry a particle is created with a finite scale along with local isotropy. This can be extended as the field is defined over all space along with the isotropy. The origin of the field always has to keep shifting after each annihilation to the replacement field's partner. This random-annihilation walk generates a location, a fuzzy location, but a location that can be described. The coordinates in time and space are now statistically independent of the original laboratory frame from where they were created. So from the laboratory frame with the physical property that allows pair-production for short intervals a localized entity can be created from something very rare an absolutely fair game of chance. This game of chance generates a statistically isolated space independent of the laboratory frame with the particle's instantaneous frame of reference tied to the current field. What is defined in this embedded space is the self-energy of the particle and this frame we call the self-reference frame. The particle's structural form in the space referenced on the particle can be generated by Taylor expanding the momentum and energy operators around r and τ of the field with the random parameter, ϵ and c/ϵ resulting in two differential equation, one for the spatial structure, $u(r)$, and one for the time dependence, $g(\tau)$, which are derived in (Wallace and Wallace, 2014) (Wallace and Wallace, 2015) and also found in summary form in appendix A.

$$\begin{gathered} E = pc \quad \rightarrow \quad E = p(r + \delta r) \quad \rightarrow \quad E = p(r + \epsilon)c \\ E = \hbar\omega \quad \rightarrow \quad E(\tau + \delta\tau) = \hbar\omega \quad \rightarrow \quad E(\tau + \frac{\epsilon}{c}) = \hbar\omega \end{gathered} \tag{8}$$

The entire concept of a point mass and point charge with their associated infinite energies vanish in this picture along with the cut offs necessary in quantum electrodynamics. What also vanishes is the single virtual photon, which cannot be supported because it will change the information content of the laboratory frame.

The student's question can be asked in a slightly different way: Is there a space where a complex random displacement exists for a field description rather than a real displacement? The independent space is defined by a representation on complex plane because primitive fields descriptions are required. A complex random displacement for a field translates to random phase shifts. It does not take very much to show that the complex random variable then has meaning not for a longitudinal field but for the transverse field.

III. STATE EQUATION IN SELF-REFERENCE FRAME

The specific particle descriptions in time and space are generated from differential equations derived by introducing a real disorder parameter, ϵ, into the massless field equation, $E = pc$, to localize the field (Wallace and Wallace, 2014) (Wallace and Wallace, 2015). The disorder parameter is also be labeled ϵ in space and $\delta\tau = \epsilon/c$ in time. The result of expanding the differential forms of the dispersion relation with the disorder parameters is a pair of differential equation for the spatial variable, r, the radial coordinate and the time coordinate, τ. The angular coordinates in spherical geometry are lost in the random behavior introduced to generate a particle description located on the instantaneous center of symmetry of the particle. The dimension of the space is defined by, n, $\gamma = E/mc^2$, $\omega_c = mc^2/\hbar$, and $\kappa = 1/\epsilon$. The resulting differential equation is from expanding the dispersion relation and referenced to the particles instantaneous center of symmetry. A quick derivation of the spatial amplitude, u(r), and the time dependence, $g(\tau)$ in the self-reference frame for Eqs. 9 and 10 are in found in Appendix A.

$$\frac{\partial^2 u(r)}{\partial r^2} + (\frac{n-1}{r} + \kappa\{1 - i\gamma\})\frac{\partial u(r)}{\partial r} - i\kappa^2\gamma u(r) = 0 \tag{9}$$

The time dependent equation can also be expanded from the the dispersion relation $E = \hbar\omega$.

$$\frac{\partial^2 g(\tau)}{\partial \tau^2} + (\omega_c + i\omega)\frac{\partial g(\tau)}{\partial \tau} + i\omega_c \omega g(\tau) = 0 \quad (10)$$

The second order spatial equation has two solutions include the hypergeometric functions ${}_1F_1$ and U where $n = 3$ and A, B, C, and D are constants (Slater, 1968):

$$u(r)_{fermion} = Ae^{-\kappa r}{}_1F_1[\frac{n-1}{1+i\gamma}, n-1, (1+i\gamma)\kappa r] \quad (11)$$

$$u(r)_{boson} = Be^{-\kappa r}U[\frac{n-1}{1+i\gamma}, n-1, (1+i\gamma)\kappa r] \quad (12)$$

The first solution represents a fermion and the second solution represents a boson both with a real mass. All densities determined from the solutions retain spherical group symmetry, *U(1)*, so that charge can be extracted. The function $u^*(r)u(r)$ is used to define the particle's static electric field if they can support a charge (Wallace and Wallace, 2015). Charge properties can be determined by an analysis of the derivative $\partial\gamma/\partial\theta$, which will define quantized charge, mass independence of charge, and the dimensional dependence of charge where θ is the argument of the complex solution $ArcTan\{Im[u(r)]/Re[u(r)]\}$. If there is no θ dependence in $u(r)$ the particle has a zero charge and cannot support an electromagnetic transition (Wallace and Wallace, 2014).

This space, the self-reference frame, is a primitive domain where no form of momentum is defined and the dynamics only refer to the relative stability of the particles. The equations are compatible with relativity through γ, which describes their behavior with different relative observers. Linear momentum, angular momentum and the magnetic moments are dynamic properties of the laboratory frame and are not part of the particle's information developed in the self-reference frame.

The importance of the self-reference frame is that as a statistically independent space it can generate the particle's self-energy. This independence is reflected in the Pythagorean sum required for the two components in the conservation of energy relation, Eq. 3, which adds the quadratic of the kinetic energy to the quadratic of the self-energy. Rather than add physical dimensions to the 3+1 space of the laboratory frame for additional particles it is possible for any particle or collection of related particles to establish an embedded private space statistically independent from the laboratory frame. This was first attempted by Dirac in 1932 (Dirac, 1932) with an introduced private time and ran into severe opposition from Pauli and Wessikopf (Pauli and Weisskopf, 1934) who used a counter argument that involved the same unfortunate Klein-Gordon equation.

A. Complex Displacement

Since mass is inversely related to the random variable ϵ to make mass complex ϵ must be made complex. By making ϵ complex it is equivalent to introducing a phase shift and this should be retarded so the transformation that will be used is found in Eq. 13 because $\epsilon > 0$ for generating a real mass. This random displacement is always positive in a spherical coordinate system as it is referenced from the instantaneous center of symmetry that is changing. Therefore for the complex displacement the relation in Eq. 13 is used.

$$\epsilon \rightarrow -i\epsilon \quad (13)$$

To transform the remaining parameters into field equations to test the freshman's conjecture about a complex mass, it is first necessary to understand how γ in the self-reference frame transforms.

$$\epsilon \rightarrow -i\epsilon \quad then \quad \gamma = \frac{E}{mc^2} = \frac{\hbar\omega_c}{\frac{\hbar}{-i\epsilon c}c^2} = -i\frac{\epsilon}{\epsilon} = -i \quad (14)$$

$$\epsilon \rightarrow -i\epsilon \quad then \quad \omega_c \rightarrow i\omega_c \quad (15)$$

For the case in the self-reference frame when the Compton wave length is set equal to the random displacement parameter, $-i\epsilon$, then $\gamma = -i$. This is one of the more important relationships derived, because it essentially enforces the quantized condition on the resultant field. In particular this is also the quantized condition for the photon energy. A particle in the self-reference frame to participate in an electromagnetic transition or the exchange of energy with an electrostatic field must be able to change γ. For a massless field either boson or fermion it is necessary that γ is a fixed complex constant that cannot vary. Therefore, the field either exists or doesn't exist and cannot be altered in any way except by absorption or emission. This constraint that $\gamma = -i$ confirms the original conjectures by Planck and Einstein that radiation is quantized.

The self-reference frame places a strict conditions on the material parameters that are defined in this independent space. Mass does not appear as a free parameter in the self-reference frame, but if the equivalent complex random displacement is applied to the particle description the student's original question can now be answered. Only the $\epsilon \rightarrow -i\epsilon$, $\kappa \rightarrow i\kappa$ and $\gamma \rightarrow -i$ are considered. The new differential equation for three dimensions in the self-reference frame for the spatial representation with a negative random complex displacement.

$$\frac{\partial^2 u(r)}{\partial r^2} + (\frac{2}{r} - 2i\kappa)\frac{\partial u(r)}{\partial r} - \kappa^2 u(r) = 0 \qquad (16)$$

The solutions can be directly written out by making the complex substitution into Eq. 11 and 12 solution with hypergeometric function rather than using spherical Bessel functions. The reason for this is to be compatible with the particle solutions and the ease of seeing how different number of spatial dimensions in the self-reference frame affect the results.

$$u_{boson} = A\ e^{-i\kappa r}\ U[1, 2, 2i\kappa r] \qquad (17)$$

$$u_{fermion}(r) = B\ e^{-i\kappa r}\ {}_1F_1[1, 2, 2i\kappa r] \qquad (18)$$

The time dependent Eq. 10 for a particle transforms to one for field by the $\epsilon \rightarrow -i\epsilon$ transformation. The time dependent field equation from the expanded dispersion relation for these fields becomes:

$$\frac{\partial^2 g(\tau)}{\partial \tau^2} + i(\omega_c + \omega)\frac{\partial g(\tau)}{\partial \tau} - \omega\omega_c g(\tau) = 0 \qquad (19)$$

The only solutions of interest are harmonic solutions of the form $e^{\pm i\omega\tau}$ that would represent a persistent and stable field. This differential equation can be examined in the general form:

$$\frac{\partial^2 g(\tau)}{\partial \tau^2} + (A + iB)\frac{\partial g(\tau)}{\partial \tau} + (C + iD)g(\tau) = 0 \qquad (20)$$

Then the only stable solutions that have a harmonic behavior are restricted to the case when in Eq. 19 the Compton frequency is the frequency of the field $\omega = \omega_c$.

$$g(\tau) = Ae^{\mp i\omega\tau} \qquad (21)$$

The complete solutions are then:

$$\phi(r, \tau)_{boson} = A\ e^{-i(\kappa r - \omega\tau)}\ U[1, 2, 2i\kappa r] \qquad (22)$$

$$\phi(r, \tau)_{fermion}(r) = B\ e^{-i(\kappa r - \omega\tau)}\ {}_1F_1[1, 2, 2i\kappa r] \qquad (23)$$

The two solutions of Eq. 16 in three dimensions produce a boson and a fermion field. The field solutions and the particle solutions can be compared in Fig. III.B, where their density functions are plotted. The original question about a complex mass first generates two fields. First is a boson with a unit density characteristic of a basic photon and then a massless elementary fermion with the charge and mass characteristics representing a neutrino. These are solutions in the self-reference frame and not in the laboratory frame where their complete structure is developed. Both solutions are of massless fields showing no perfered local structure. This was forced by $\gamma = -i$ being fixed complex constant. Any other values of γ produced divergent solutions that are not valid. Divergence here means that the density functions grow larger with increasing r, which is neither the property of a physical realizable particle or field. Fixing γ for massless field also insures the independence of the speed of light in any reference frame. This restriction on γ is a requirement for the quantization of the field for both the photon and neutrino. The detailed behavior as a function of γ are found inand for their associated anti-particles (Wallace and Wallace, 2014). The fermion field had zero charge because the wave function, *u(r)*, has no variation in its complex argument. The solution, *u(r)*, from Eq. 18 for the fermion field is purely complex when $B = 1$. The time dependent solutions are simple harmonic solutions with a frequency, $\omega_c \rightarrow ic/\epsilon$ originally $\omega_c = c/\epsilon$ and the solution in Eqs. 22 and 23 represents a persistent and stable field.

In the self-reference frame the harmonic time dependence of a stable entity that starts with a single time dependence when the frame is created with no previous history, as all entities whether a particle or a field come with their own clocks, via their time dependence, and are essentially isolated by the statistical independence of the space in which they were generated. The only exception is when two or more particles share the same clock either from being created at the same instance or interacting with one of two fields or particles that were created as a pair. This behavior is important for understanding entanglement. One requirement for relativity is a measurement scale and a time base and these conditions are satisfied for each individual particle.

B. Particle and Field Density

In the total wave function in the self reference frame $\phi(r, \tau) = u(r)g(\tau)$ the time dependence being of the form $e^{-i\omega\tau}$ becomes a constant factor in the probability density function. The particle density in the self-reference frame in three dimensions is given by the expression $u^*(r)u(r)r^2$. The core of density $u^*(r)u(r)$ in the case of a massive fermion is proportional to the static electric field and removes the $1/r^2$ singularity of the point electron at its center of symmetry (Wallace and Wallace, 2015). In the case of the massive boson the properties of weak charge result and the description is found in (Wallace and Wallace, 2014). For the massless fields the boson density is a constant as it is for the photon field. However, for the fermion field it has an oscillatory behavior as shown in Fig. III.B. It is the energy dependent oscillatory character of the density function that is of primary interest as it reduces the particles interaction cross section.

$$u^*_{neutrino}(r)u_{neutrino}(r)r^2 \sim Sin^2\kappa r \tag{24}$$

The mean value of the Sin^2 term is exactly one half. This behavior in the spatial portion of the wave function is unique among particles and will lead to a reduction in detected sensitivity by exactly 50% in measured data whether from solar or reactor generated electron neutrinos.

The extensive literature on the neutrino-cross section as a function of energy that result are dynamic calculations at a level above of the density calculation for the neutrino in the self-reference frame (Formaggio and Zeller, 2012). The kinematic models do not involve the structure of the particles themselves, only their bulk properties and allowed interactions. It is not necessary to involve the specific mechanisms for the energy dependent calculation of cross-sections, because the correction being introduced will affect the neutrino across its entire energy range uniformly.

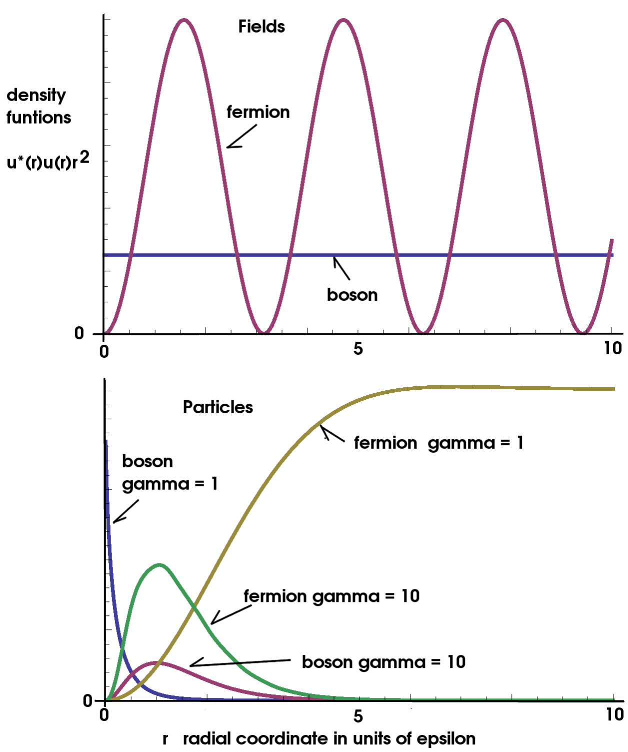

FIG. 1 **Density functions of fields and particles in the self-reference frame. The individual density scales are arbitrary so the functions separate.**

IV. BOREXINO DATA

The ν_e detection method is to monitor the scattering process (ν_e, e^-) in a large liquid scintillator (Derbin and group, 2016). The Borexino analysis assumes the standard solar model chain of coupled fusion reaction and decays generates a distribution of isotopes that is accurate from an end point analysis of star surface chemistry. To do this analysis one has to assume that knowledge of all possible reactions are included and accurately accounted for including the dependence on the distribution of material through out the sun as a function of depth and temperature. The second assumption is that the calculated kinetic neutrino cross section is assumed to be correct, because there is a good understanding of the weak processes. The strength of the analysis and experiment is that most of the activity with the ν_e occurs for processes that can be individually isolated. The data of interested is presented in Table I.

TABLE I **Probability of solar neutrino survival data from Borexino (Derbin and group, 2016). The *pp* and the ^8Be have continuous neutrino spectra down to zero energy. The *pp* process is the dominant process but with a small detection cross-section making it more difficult to resolve at low energy. The mean on the unweighted sum of the survival probability is .49 ± .11**

Energy	Process	**Mean**	**Low**	**High**
.3 - .4 MeV	$^1H + ^1H \to ^2H + e^+ + \nu_e$	.64	.52	.76
.89 MeV	$^1H + ^1H + e^- \to ^2H + \nu_e$	.62	.47	.79
1.5 MeV	$^7Be + e^- \to ^7Li + \nu_e$	.52	.46	.58
3 - 18 MeV	$^8B + e^- \to ^8Li + \nu_e$	.38	.27	.51
5 - 18 MeV	$^8B \to ^8Li + e^+ + \nu_e$	.31	.22	.43
.3 - 18 MeV	Means (theory .5)	.494	.388	.614

If the neutrino density function from Eq. 24 is correct there will be a factor multiplying the flux measurements which is *.5* and the average of the unweighted five process is .49 ± .11. This unweighed result is very close to the expected decrease computed for the reduction produced by the neutrino density function. Because of the manner of experimentally isolating individual components the best comparison that can be made is an unweighted mean. Tying the deficit of neutrinos to a reduction in neutrino flux rather than a reduction in detector sensitivity across a significant energy range leads to a false conclusion about ν_e mass. The data indicates the solar ν_e is most likely a massless field. This leaves the problem of transformation of flavors in neutrinos as an open question and affects corrections to any proposed anomalous mass for ν_μ and ν_τ.

V. REDUCED SPATIAL DIMENSION IN SELF-REFERENCE FRAME

The solutions of Eqs. 9 and 16 are also valid for dimension less than three. There is some value in these solution since for the fermions with mass they generate fractional charge of 2/3 and 1/3 for particles in two and one spatial dimensions (Wallace and Wallace, 2014). The field density function in two and one dimensions are shown from the same analysis of Eq. 16 in Fig. V and and Table II. In three dimension and one dimension the fermion quantized field associated solutions show zero mass behavior with no local structure. That is not the case in two dimensions where the density function does show local structure. Upon the oscillating density function there is a modulation very similar to a three dimensional massive elementary fermion in its relative rest state or the two dimensional boson field. This structure showing a mass like behavior differs from that found for a massive fermion because it is only define for the fixed case where $\gamma = -i$. This characteristic is different than inertial mass of a particle where $\gamma \geq 1$, where γ can scale over a range and is not fixed. The two dimensional fermion is still a field moving at the speed of light but having some of the characteristic of a rest state zero charge fermion with mass or a 2D massless boson field. The two dimensional solution might be associate with a component that can be used to produce a three dimensional ν_μ muon neutrino. The one dimension case has a density function which has no spatial oscillations and therefore no reduction in cross section. The one dimensional case also might be a basis state that can be used to assemble ν_τ.

TABLE II **One and two dimensional density functions.**

Dim.	Boson density	Fermion density
2	$U[\frac{1}{2},1,2i\kappa r]U[\frac{1}{2},1,-2i\kappa r]r$	${}_1F_1[\frac{1}{2},1,2i\kappa r]{}_1F_1[\frac{1}{2},1,-2i\kappa r]r$
1	1	1

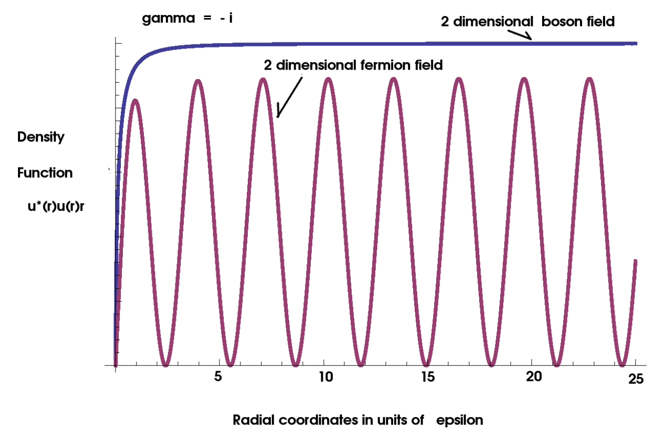

FIG. 2 **Density functions of 2D fields in the self-reference frame. The dip makes the two dimensional density functions different from the three dimensional density function. The individual density scales are arbitrary.**

VI. DISCUSSION

The first assumption made is that the uncertainty principal operates in time and space in the laboratory frame to generate a representation in the self-reference frame. The second assumption is that the elementary entities of the self-reference frame are transverse and longitudinal fields. The third assumption is that energy conservation is maintained for any field interactions. The complex plane becomes the representation space because the primitive entities are scalar harmonic fields. This is sufficient information to generate the stable elementary entities we detect. Particles, mass, charge, symmetries, isotropy, and quantized fields then become derived properties and not properties that have to be assigned. Relativity is satisfied in the self-reference frame where the particle's self-energy is generated and the massless fields are quantized.

The compact extraction of three stable and well behaved starting representations for a stable elementary fermion, electron, a stable quantized massless boson field, photon, and a stable massless zero charge fermion field, ν_e electron neutrino, from an embedded statistically independent space solves a problem of directly integrating relativity into quantum mechanics at a level below that of the dynamics of the fields and particles themselves. This interpretation of quantum mechanics for the generation of material properties runs counter to the restrictions of the Copenhagen school of quantum mechanics and allows solving a number of long standing problems.

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Jack Steinberger in an elementary physics course to a group of freshmen stated he had forgotten classical mechanics and it was only possible for him to teach quantum mechanics. This made perfect sense as quantum mechanics cannot be derived from classical mechanics. After introducing the Schrödinger equation a student named Peter Landesman asked "What would be the result if mass in the Schrödinger would be a complex number?" Also he introduced the question about how primitive longitudinal fields in physics are poorly understood. This year is the 50th anniversary of that class.

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Appendix A: Derivation of State Equations in Self-Reference Frame

Starting with the dispersion relation for a massless field in laboratory frame (Wallace and Wallace, 2015):

$$E = pc \tag{A1}$$

The spatial dependent equation will be derived first where $u(\mathbf{x})$ is the spatial dependent function.

$$i\hbar c\nabla u(\mathbf{x}) = Eu(\mathbf{x}) \tag{A2}$$

The scale of uncertainty in space, ϵ, enters the spatial equation as a random offset that is greater than zero. In the spatial differential equation becomes a second order differential equation.

$$\begin{aligned} u(\mathbf{x}) &\rightarrow u(\mathbf{x}+\epsilon) \\ u(\mathbf{x}+\epsilon) &= u(\mathbf{x}) + \epsilon u'(\mathbf{x}) \\ \nabla u(\mathbf{x}+\epsilon) &= \nabla u(\mathbf{x}) + \epsilon \Delta u(\mathbf{x}) \end{aligned} \tag{A3}$$

$$\{\nabla u(\mathbf{x})\}_r \rightarrow \frac{\partial u(r)}{\partial r} = u'(r) \tag{A4}$$

$$\{\Delta u(\mathbf{x})\}_r = \frac{\partial^2 u(r)}{\partial r^2} + \frac{n-1}{r}\frac{\partial u(r)}{\partial r} = u''(r) + \frac{n-1}{r}u'(r) \tag{A5}$$

$$u''(r) + (\frac{n-1}{r} + \kappa\{1-i\gamma\})u'(r) - i\kappa^2\gamma u(r) = 0 \tag{A6}$$

Where $\kappa = 1/\epsilon$, $\gamma = E/mc^2$, and n is the number of spatial dimensions.

a. Time Dependence

The time dependent equation for $g(\tau)$ will be similarly derived from the massless dispersion relation.

$$-c\hbar k g(\tau) = -\hbar\omega g(\tau) = i\hbar\frac{\partial g(\tau)}{\partial \tau} \tag{A7}$$

Similarly in the first order time dependent equation the uncertainty in time enters as $\delta\tau = \epsilon/c$.

$$g(\tau+\delta\tau) = g(\tau) + \delta\tau\frac{\partial g(\tau)}{\partial\tau} + \ .\ . \tag{A8}$$

$$\frac{\partial g(\tau+\delta\tau)}{\partial\tau} = \frac{\partial g(\tau)}{\partial\tau} + \delta\tau\frac{\partial^2 g(\tau)}{\partial\tau^2} + \ .\ . \tag{A9}$$

$$i\omega(g(\tau) + \delta\tau\frac{\partial g(\tau)}{\partial\tau}) = \frac{\partial g(\tau)}{\partial\tau} + \delta\tau\frac{\partial^2 g(\tau)}{\partial\tau^2} \tag{A10}$$

$$\frac{\partial^2 g}{\partial\tau^2} + (\omega_c - i\omega)\frac{\partial g}{\partial\tau} - i\omega\omega_c g = 0 \tag{A11}$$

Appendix B: Extended Wave Equation

It is simple to derive something functional to replace the Klein-Gordon equation that conserves energy and compatible with relativity as a second order wave equation in the laboratory frame (Wallace and Wallace, 2014). The energy operator, which is a first order time derivative, is taken as the total energy less the self-energy. This compatible both with the Schrödinger equation and the Dirac equation and does not violate the quadratic relativistic conservation of energy condition (Fermi, 1961).

$$i\hbar\frac{\partial}{\partial t} \rightarrow E - mc^2 \tag{B1}$$

Using the momentum operator and the correct energy operator equation 3 is converted into the resulting differential equation, which has the form of the telegraphers equation, with a complex term multiplying the first order time derivative rather than a real term as found in the Klein-Gordon equation.

$$\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \frac{2mi}{\hbar}\frac{\partial}{\partial t} = 0 \tag{B2}$$

Solution is spherical coordinates for a particle and field respectively in the laboratory frame reference to center of symmetry of the entity.

$$\phi_{particle} = A\frac{e^{-\kappa r + i\omega t}}{r} \tag{B3}$$

$$\phi_{field} = B\frac{e^{i(\kappa r-\omega t)}}{r} \tag{B4}$$

The solution particle has a scale of ϵ which rolls off and the field has the characteristic wavelength ϵ. The solutions of particle wave function are rest frame solutions of the laboratory frame and differ from the dynamic solutions of both the Schrödinger and Dirac equations.

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