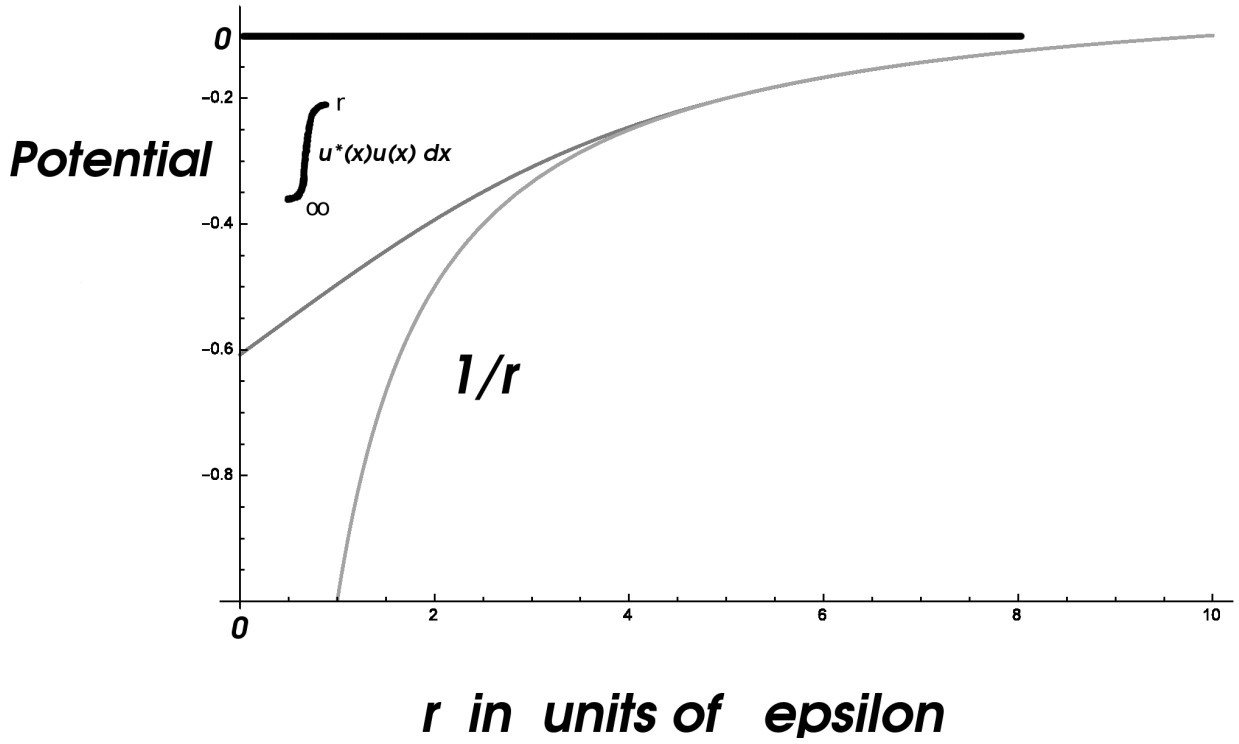
Electrostatics



How accurate is Coulomb's law? After 231 years you would think the answer to that question would be known. The existence of quantum electrodynamics suggests that Coulomb's law is an approximation. Actually its a rather poor approximation when working at the Compton wave length of a particle.

In the enclosed paper, which expands on the work on charge done in our recent book, *The Principles of Matter amending quantum mechanics*, www.castinganalysis.com/abook we demonstrate how to construct the electrostatic field for an electron. The effort can be verified by examining the corrections to the ionization data of single electron atoms.

Electrostatics *

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Abstract.
Quantum mechanics should be able to generate the basic properties of a particle. One of the most basic properties is charge and the associated electrostatic electric field. Electrostatic force is a fundamental characteristics of a charged fermion and should have its nature described by the fermion's structure. To produce the particle properties require two spaces that define both their dynamics and their base structure. Relativity and the conservation of energy dictate how these two separate spaces are connected and the differential equations that describe behavior within these two spaces. The main static characteristic of an elementary fermion are mass and charge. Mass represents a scale measure of the fermion and it appears that charge results from the detailed structure of the fermion, which must merge into the electric field description of Maxwell. Coulomb's law is a good approximation for large distances, but it is a poor approximation at dimension on the order of a particle's Compton wavelength. The relativistic description of the fermion in its own frame of reference contains the information required for producing the electrostatic field over all space without a singularity as a source. With this description it is possible to understand the first order correction to the ionization energy of hydrogen. The role of nuclear effects on ionization energies can now be better defined for nuclei heavier than hydrogen.

Introduction

This work on electrostatics can be traced directly to trying to understand the limits of the spherical flat bottom potential problem from quantum mechanics while trying to model the diffusion of a proton in niobium [1]. Understanding how a proton is restrained and diffuses in niobium is necessary to understand the effects of hydrogen on the superconducting properties of niobium for improving the efficiency of running accelerator at its design limit. The simplest 3D bound state problem when examined in the limit of weak binding has a set of three low level solutions. Trying to explain the existence of these three solutions eventually led to an entirely different way of integrating relativity into quantum mechanics. At that time we became aware of accelerator measurements at Jefferson Lab of short-range correlation data generated in the accelerator we were concerned with in terms of hydrogen contamination. This paper continues that research track and finds some very old experimental data on ionization measurements as a source for nuclear properties of a few light nuclei. Ionization data rather than spectroscopic transition data is more revealing of transient nuclear properties of some stable light nuclei. One of the major research goals at Jefferson Accelerator Laboratory is to understand the nature of the short range correlation behavior and the approach introduced here will add to that data. Our efforts are a good argument against the compartmentalization of research between the building and the use of accelerators because the problem in both areas require all the physics knowledge available to make progress. Physics is a rather compact subject and there will always be considerable overlap. High energy physics faces a sterility problem because it is very isolated from of the rest of science [2]. The theories that it has used to layout future research are highly questionable. To make real progress there must be a consistency in understanding across all energy scales and that requires not effective field theories but field theories that are accurate across those wide scales, which must include gravity. The recent relative energy dependent measurements of gravity by using the Compton edge accelerator data makes a good start on taking on this broader approach to high energy physics [3].

*J.P. Wallace & M.J. Wallace, Electrostatics, in *Science and Technology of Ingot Niboium for Superconducting Radio Frequency Applications* ed. G. Myneni, AIP Conf. **Vol 1697**, pp 040004-1-14, AIP Melville NY 2015, (some minor changes in notation were made for clarity and typographical errors were removed from the referenced version)

Static Properties

Quantum mechanics has supplied no mechanism for the source of a charged particle's electric field. Historically this problem goes back to the debate on quantum mechanics between N. Bohr and A. Einstein centered on the statistically random nature of quantum mechanics. Einstein was searching for a realism he felt existed, which the statistical basis of quantum mechanics might suppress. Whereas, Bohr was convinced that only the measured phenomenon are real and there was nothing more to be gained at pondering the details behind the statistical basis of quantum mechanics. A parallel debate between Dirac and Pauli was also never settled about how to represent relativity in a quantum context [4] [5] concerning an important point about time, which is tied to the original debate between Bohr and Einstein. These were important arguments and the difficultly was in understanding that time and space are coupled statistically.

Experimental data and the simple experience of working with a quantum object on a large scale [6] [7] was essential in taking the 1932 arguments of Dirac, which concerned more than time, but how relativity should enter into quantum mechanics to generate particle properties. The problem at the base of these arguments is whether there was more than one space in which to apply quantum mechanics. A space to generate properties and a space to compute dynamics. Why and how should two such different entities exist? The argument for this division came from considering the very simple model of a spherical potential well acting on a field and asking the question, what is the minimum potential required to generate a bound state to localize a field? That can be computed if the local interaction is attractive and it has three allowed solutions which generated a scale for the entities. This is the first step in defining a primitive boson [1]. For a locally repulsive interaction this would describe an elementary feature with properties associated with a fermion. The scale that is generated is a measure of uncertainty in the description of the original field. This uncertainty is defined by a single parameter, ϵ, as the scale. But in time there is also an uncertainty generated that is $\Delta\tau = \epsilon/c$. Once this point was made it was apparent there was enough information to define a space where a particle generates its properties that is statistically independent from the laboratory frame. This space is called the self-reference frame. Standard quantum field theories are dynamical theories analogous to classical mechanics. This new space is not the same kind of dynamical space used in mechanics but it describes the actions on a field which generate particle properties. Momentum is not a part of this new space.

Identifying the particle's self-reference frame allows a quantum particle to generate its properties, which are realized by measurement in the laboratory frame. The statistical basis of Bohr is preserved but the real properties that Einstein wanted are generated from this basis. What was then necessary was a description of a particle in this new frame of reference and that is a straight forward construction. The relativistic energy conservation requirement will hold in both reference frames though it will be altered by what is allowed to be defined in each frame. Principally mass is not defined as a separate property in the self-reference frame but is generated in that frame so the energy conservation is transformed.

$$(E - mc^2)(E + mc^2) = p^2c^2 \implies E^2 = p^2c^2 \tag{1}$$

In the self-reference frame equation 1 can be factored into a product of two simple dispersion relation $E = pc$. From these a separate spatial and time dependent second order differential equations can be constructed by imposing the disorder parameter for space and time on the particle's spatial $u(r)$ and time $g(\tau)$ dependent particle descriptions [8] [9]. A derivation of the resulting differential equations is in the appendix. The product of these individual descriptions then construct the particle's state function in the self-reference frame. Experimentally these analytical representations were confirmed for a massive boson in the previously cited references. But massive bosons are exotic things, whereas electrons and nucleons are more commonly dealt with. One of the simplest forces is electrostatics and this should be something that is generated along with charge in the self-reference frame for an electron.

Self-Reference Frame Representation

To generate the structural characteristics of a particle, the particle must be defined in its own frame of reference independent of the laboratory frame. There is no analogous transformation between the two frames similar to a Galilean transformation as the two frames are statistically independent. The simplest way to introduce the relation of the self-reference frame to the laboratory frame is in Table I that lists properties, characteristic and how they differ. This particle space allows the elementary fermion to generate an electrostatics field. The quantization of charge can be derived by considering the allowed transitions that can occur from the self-reference frame as a function of the number of spatial dimensions in the laboratory frame. Boson charge or the weak charge and charge quantization are derived from considering these transitions and are discussed in detail in reference [9].

TABLE 1. Self-reference frame dependent features for a free unbound particle, ref. [9]

Property	**Description**	**Comment**
Lab. Frame	$\mathbf{R}^n$	with time $\mathbf{R}^3 + 1$
Self-Ref. Frame	$\mathbf{C}$	with time $\mathbf{C} + 1$
State Function	$\psi(r,\tau) = u(r)g(\tau)$	defined in self-ref. frame
ϵ	random scale variable	generates the self-reference frame
γ	relative energy	$\gamma = E/mc^2 \quad \gamma \geq 1 \ \ or \ \ \gamma \leq -1$, real
κ	inverse scale $1/\epsilon$	$\kappa = mc/\hbar$, real for massive particles
ω_c	frequency	$\omega_c = mc^2/\hbar$ real
r	radial variable	real $r \geq 0$
τ	time variable	real $\tau \geq 0$
n	dimension	$r^n \sim Volume$
$r = 0$	center of symmetry	a random location in the lab frame
$u(r)$	spatial function	complex function, Amp. $\|u(r)\|$ & Arg. θ
$u^*(r)u(r)r^2$	density function	measured for a boson
$u^*(r)u(r)$	U(1) symmetry	proportional to electrostatic field
Sign[n, γ]	field sign function	$(-1, \ \gamma \geq 1) \ \ or \ \ (+1, \ \gamma \leq -1) \times \frac{n}{3}$
dynamics	lab frame only	boson $\rightleftharpoons$ fermion in self-ref. frame
angular momentum	lab frame	not defined in self-reference frame
mass, charge, **E** & **B**	lab frame	not defined in self-reference frame
gravity	lab frame	not defined in self-reference frame
Lorentz trans.	lab frame	not defined in self-reference frame

The differential equations that represent the spatial structures $u(r)$ and the time dependence $g(\tau)$ in the particle's self-reference frame are derived from the factored coordinate independent energy conservation relation in equation 1 [9]. This analysis defines a particle's locality that differentiates a particle from a field. The only parameters in the frame of reference of the particles are γ from relativity $E = \gamma mc^2$ and $\kappa = mc/\hbar$ the inverse of the Compton scale for a particle of mass, m, where c is the speed of light and $\hbar$ is Planck's constant. This analysis yields two differential equations one for space and one in time.

$$u''(r) + (\frac{n-1}{r} + \kappa\{1 - i\gamma\})u'(r) - i\kappa^2\gamma u(r) = 0 \tag{2}$$

The two solutions in 3-dimensions spatial equations are given in equation 3 and 4. These two solutions represent a boson and fermion which are parametrized in terms of their relative energy $|\gamma| \geq 1$ and the scale of the particle's core $\epsilon = 1/\kappa$.

$$u_{boson}(r;\gamma,\kappa) = Ae^{-\kappa r}U[1 - i\gamma, 2, (1 + i\gamma)\kappa r] \tag{3}$$

$$u_{fermion}(r;\gamma,\kappa) = Be^{-\kappa r}{}_1F_1[1 - i\gamma, 2, (1 + i\gamma)\kappa r] \tag{4}$$

The solution for the fermion function depends on the confluent hypergeometric function ${}_1F_1$ and the boson is described by the solution containing the U function. The solutions in the self-reference frame for the fermion function have both the characteristics of a local center of symmetry and the wave characteristics of a field over all space. The wave-particle duality is built into the description from the beginning in the self-reference frame. For relativity, both a scale and clock are required and these spatial solutions supply the particle's scale.

The time dependent equation for the state function which is also derived from equation 1, has a very simple solution set.

$$\frac{\partial^2 g}{\partial \tau^2} + (\omega_c \mp i\omega)\frac{\partial g}{\partial \tau} \mp i\omega\omega_c g = 0 \tag{5}$$

For the equation with, $\omega = (\gamma - 1)\omega_c$ the positive terms there are two solutions where ω_c is the Compton frequency, see Table 1.

$$g_+(\tau) = Ae^{-i(\gamma - 1)\omega_c\tau} + Be^{-\omega_c\tau} \tag{6}$$

For the equation with negative terms there are also two solutions.

$$g_-(\tau) = Ae^{i(\gamma - 1)\omega_c\tau} + Be^{-\omega_c\tau} \tag{7}$$

These solutions supply the particles internal clock with the time dependence due to the particles relative kinetic energy. The time and distance scales are coupled through the particles self-energy. Note that the time dependent version of these solution for a massless entity have no decay modes and therefore no virtual existence [9].

The spatial and time dependent equations of a particle in the self-reference frame supply both the clock and ruler required for the laboratory frame and essential for special relativity. The time dependent relation's solution only has meaning for both solutions if $\tau > 0$. The decaying solution has to reappear elsewhere, and that problem involves the relative stability of both the fermion and boson states. Dynamics in the self-reference frame is a study in relative stability as particles can transform into more stable forms. As the elementary boson and fermion are the only two things that populate the self-reference frame the dynamics are limited to the relative stability for these two forms.

The representation for the elementary massive fermion and boson allows their antiparticle to be represented as their complex conjugate. This is equivalent of allowing γ to be replaced by $-\gamma$ and representing the negative energy solutions. The spatial density distribution of the particle is supplied by the product, $u^*(r)u(r)r^2$, represents a measurable property that can be detected.

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The division between electrostatics and electrodynamics is not one of semantics, it is a major division of two very distinctly different mechanisms that result in measurable forces. Electrodynamics is understood a little better than electrostatics in terms of its foundation. There is a separation that Michael Faraday pointed out in the mid nineteenth century and has really not been explained any better since that time [10]. The current way to describe electrostatics is to invoke a virtual photon [11]. This appears to be an unnecessary complication to describe a contact interaction. Attempts to get around this problem with two photon exchange [12] also have not captured the physics required to generate the electrostatic field.

To explore the idea of what produces the static electric field it is useful to plot the product function for a stable fermion in the self-reference frame $\psi^*\psi$, which reduces to $u^*(r)u(r)$ that is time independent. For the fermion it was noticed that this function away from the center of symmetry falls off as $1/r^2$. This is identical to the fall off in the static electric field in the laboratory frame for an electron. Since charge does not alter the curvature of the laboratory frame and the field that it generated might be associated directly with the product $u^*(r)u(r)$ where its symmetry is determined from the restriction to a radial dependence.

The spatial portion of the fermion state function in three dimensions that will be used to generate the electric field and potential is:

$$u(r) = Ae^{-\kappa r}{}_1F_1[1 - i\gamma, 2, (1 + i\gamma)\kappa r] \tag{8}$$

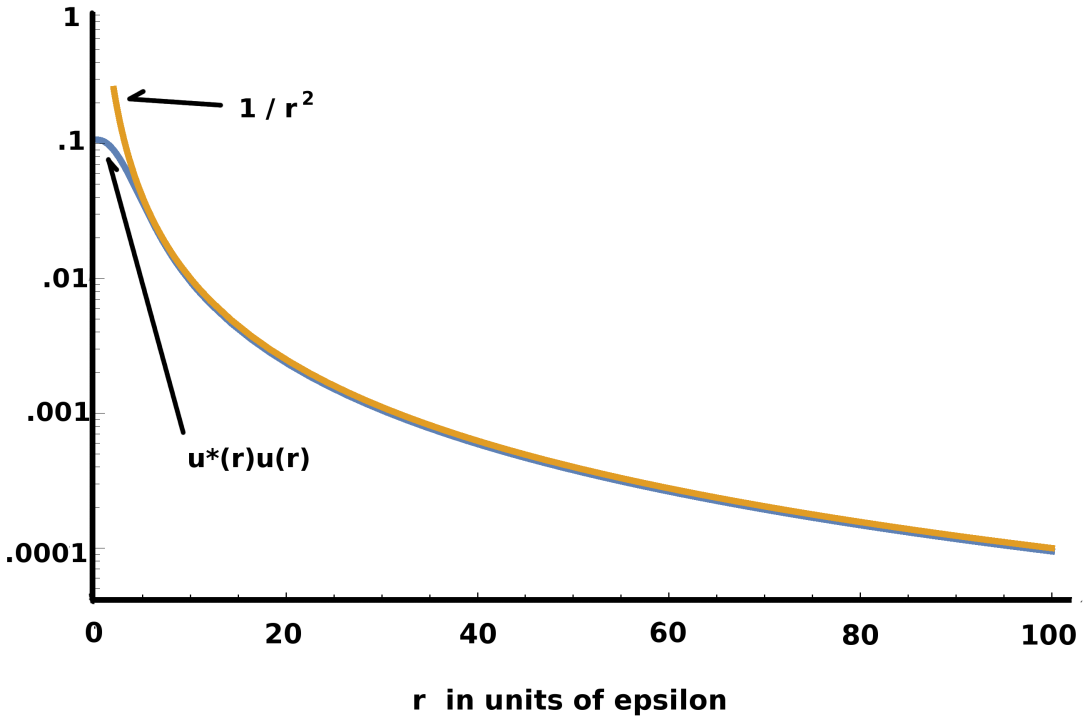

FIGURE 1. Electric Field Comparison: The $1/r^2$ fall off in the fermion, u^*u function at large distances is characteristic of the electric field from a point charge. Where ϵ, epsilon, is inversely proportional to mass with $\epsilon = \hbar/mc$ and for the electron it is approximately 3.86×10^{-13} meters.

The trial function describing the spatial dependence of the electrostatic electric field is:

$$\mathbf{E}(r) = \frac{e}{4\pi\epsilon_o} u^*(r)u(r)\,\widehat{\mathbf{r}} \tag{9}$$

which in the limit as $r \to \infty$ the normalization constant can be computed with a field dependence is $1/r^2$ using the series expansion for ${}_1F_1$ at large arguments [14], where Γ is the gamma function.

$$\lim_{r\to\infty} AA^* e^{-2\kappa r}\,{}_1F_1(r)\,{}_1F_1^*(r) = \frac{AA^*}{\Gamma[1-i\gamma]\Gamma[1+i\gamma](1+\gamma^2)e^{-2\gamma\ ArcTan(\gamma)}\kappa^2} \times \frac{1}{r^2} = \frac{1}{r^2} \tag{10}$$

This allows setting A^*A equal to the denominator $\Gamma[1-i\gamma]\Gamma[1+i\gamma](1+\gamma^2)e^{-2\gamma\ ArcTan(\gamma)}\kappa^2$ to normalize the result to a simple $1/r^2$ in equation 10 then the electric field and potential are determined.

$$\mathbf{E}(r) = \frac{e}{4\pi\epsilon_o}\Gamma[1-i\gamma]\Gamma[1+i\gamma](1+\gamma^2)e^{-2\gamma\ ArcTan(\gamma)}\kappa^2 e^{-2\kappa r}\,{}_1F_1[1-i\gamma,2,(1+i\gamma)\kappa r]\,{}_1F_1[1+i\gamma,2,(1-i\gamma)\kappa r]\,\widehat{\mathbf{r}} \tag{11}$$

$$v(r) = \int_\infty^r u^*(r')u(r')dr' = \int_\infty^r \mathbf{E}(r')\bullet d\mathbf{r}' \tag{12}$$

The validity of $v(r)$ potential can be explored by using the function in computing the matrix elements between states of the hydrogen atom when the $1/r$ point charge electrostatic potential is subtracted. This difference in the potentials will be represented as δV in the case of atomic hydrogen.

$$\delta V(r) = -\frac{e^2}{4\pi\epsilon_o}\{\frac{1}{r} - v(r)\} \tag{13}$$

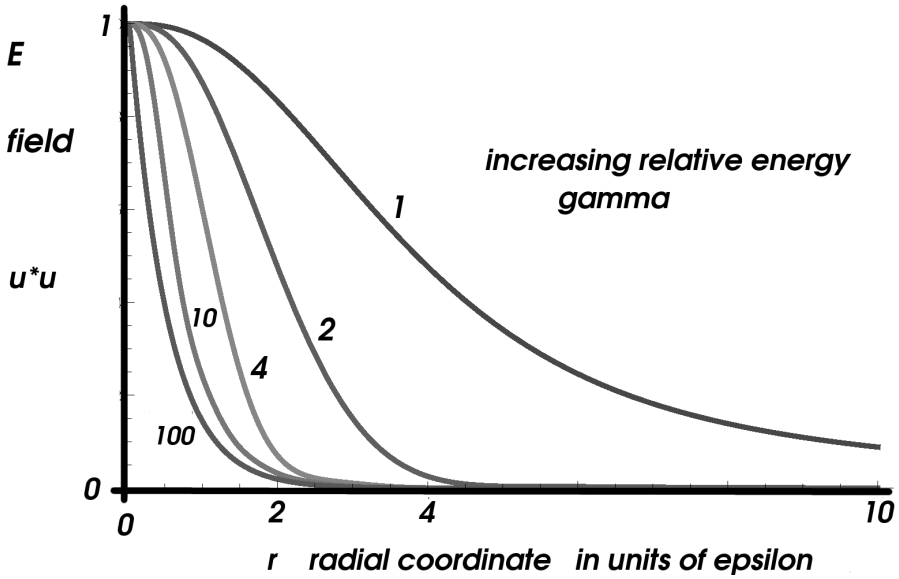

FIGURE 2. Electric Field $u_f^*(r,\gamma)u_f(r,\gamma)$ comparison as a function of relative energy, γ. The contraction of the field occurs with increasing γ.

This is not a small potential change because of the removal of the singularity at the origin, but it is restricted to a small volume which lessens its impact on atomic properties but not on nuclear properties.

Nuclear Potentials

To improve the relevance of the calculation because of the charge radius problem [13] requires defining a potential for the nuclear charges in terms of $u(r)$ for the charged nucleon. To differentiate between the different potential sources: electron, proton and muon the function $u_e(r)$, $u_p(r)$ and $u_\mu(r)$ are used respectively. The potential then becomes:

$$v_i(r) = \int_\infty^r u_i^*(r')u_i(r')dr' \quad where \;\; i = e,\; p,\; or\; \mu \tag{14}$$

Then the error potential for hydrogen like atom, proton-muon or positronium can be computed the particle's potential.

$$\delta V_{i-j}(r) = \frac{e^2}{4\pi\epsilon_o}\{v_i(r) - v_j(r)\} \tag{15}$$

These error potentials are useful in evaluating the magnitude of the contribution for correcting the point charge nuclear potentials.

Charge Density

The charge distribution can also be defined from the electric field. A trial definition for the electric field, $E(r)$ is:

$$\mathbf{E}(r) = \frac{e}{4\pi\epsilon_o}u^*(r)g^*(\tau)u(r)g(\tau)\widehat{\mathbf{r}} = \frac{e}{4\pi\epsilon_o}u^*(r)u(r)\widehat{\mathbf{r}} \tag{16}$$

Using Gauss's law the charge density can be computed.

$$\nabla \bullet \mathbf{E} = \frac{\rho(r)}{\epsilon_o} \tag{17}$$

$$\rho(r,\gamma) = \frac{e}{4\pi r^2}\frac{\partial}{\partial r}\{r^2u^*(r,\gamma)u(r,\gamma)\} \tag{18}$$

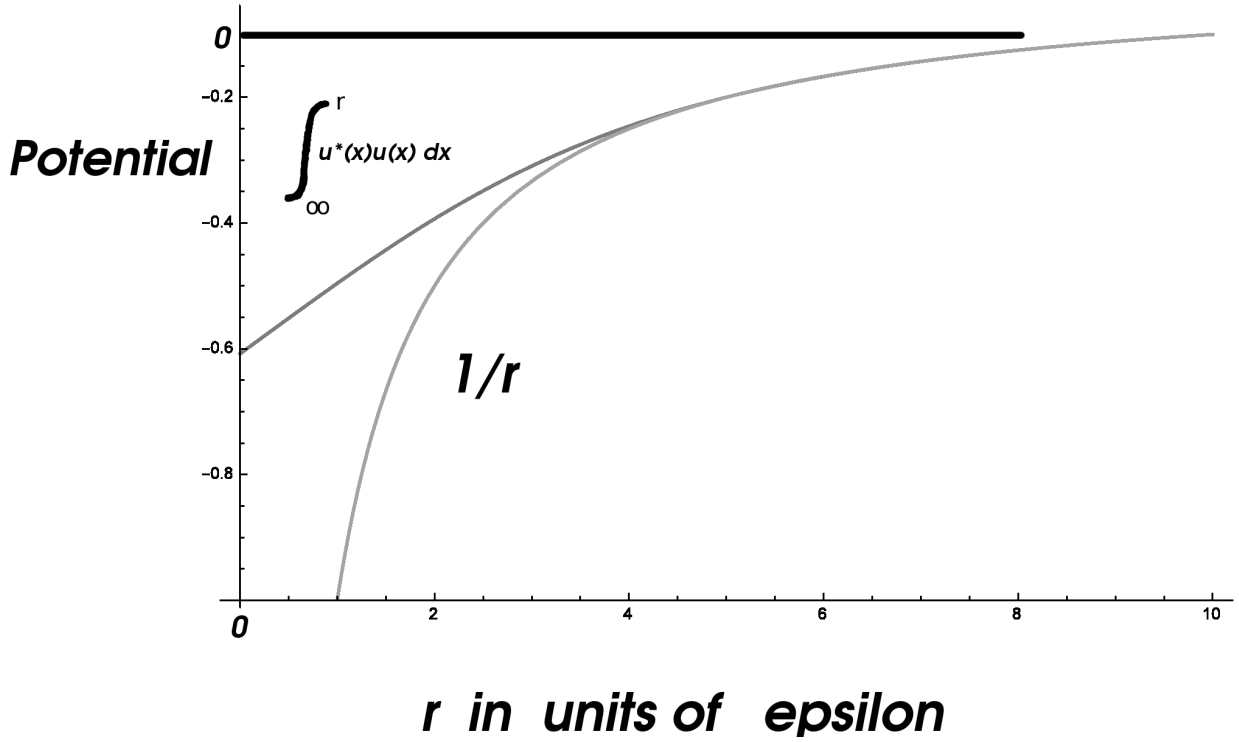

FIGURE 3. The $1/r$ fall off in the fermion function at large scale shows the same behavior as that of a point charge. At short distances the function goes to a terminal finite value. This implies that the electrostatic interaction is a contact interaction between charges over their volume. The contact is that of the combined fields that determine the local energy density and its resulting action on the interacting particles center of symmetry producing a net force.

$$\rho(r,\gamma) = \frac{e}{4\pi}\{\frac{2}{r}u^*u + u^{*'}u + u^*u'\} \tag{19}$$

The charge density is shown in Figure 4. Having the charge and the electric field directly generated from the particle's density function, which is distributed over spaces, eliminates the need for virtual photon as there can be a contact interaction between neighboring fields removing the need for an intermediate agent.

The charge density is localize to a small volume unlike the particle density, $u(r)^*u(r)r^2$, which can be spread over all space. The total charge can be examined by electromagnetic scattering, but the small volume over which it is defined limits the information that can be acquired in any single experiment. The errors in high energy measurements will be large compared to the resolution required to map the structure. It is using low energy experiments that will yield the more accurate data about the fermion structure.

First Order Corrections & Data

Even though the proposed particle's derived electric field removes a singularity in the self-reference frame for charge, the bulk of the charge is limited to a small volume. The most accurate method to determine the impact of the distributed charge to the atomic hydrogen energy levels is to solve the Schrödinger equation with the correct potential by replacing the $1/r$ potential with one dependent on the fermion state function, $u(r)$ as it determines $V(r)$ from equation 12.

$$-\frac{\hbar^2}{2m}\nabla^2\phi + V(r)\phi = E\phi \tag{20}$$

It is not apparent how to reduce this equation to an analytical solution. Because the changes in potential are localized about the origin on a scale much less than that of the Bohr radius it is easier to compute the first order perturbation

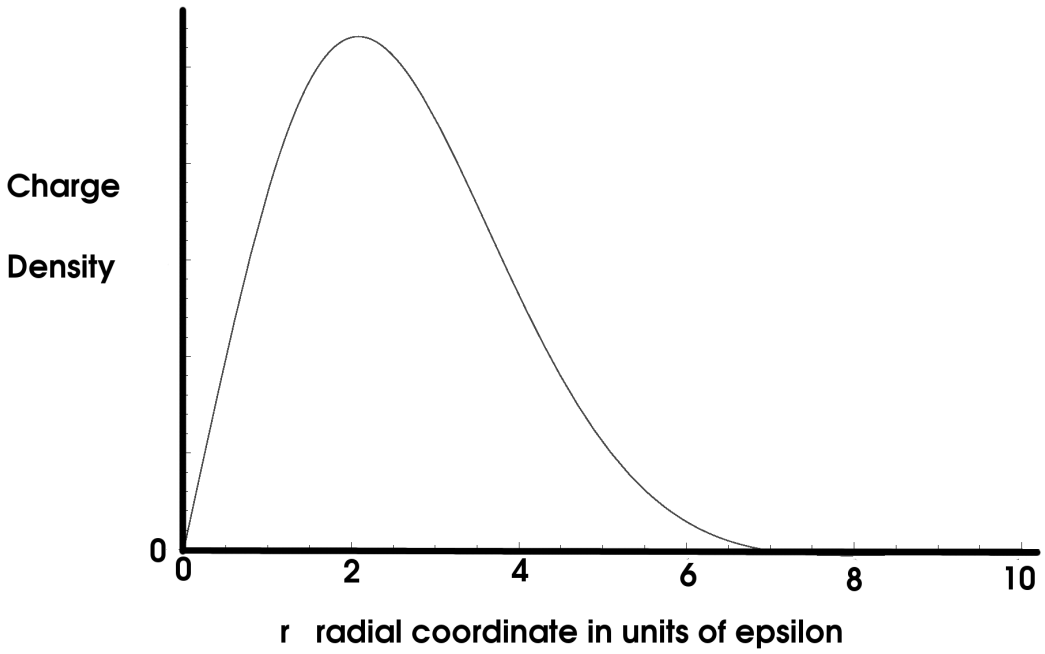

FIGURE 4. Charge Density: The charge density plotted as $\rho(r)r^2$ as computed in equation 19. The radial coordinate is in units of ϵ where $\epsilon = 1/\kappa$ and $\gamma = 1$.

correction from the Schrödinger equation using the standard proton-electron point charge electrostatic approximation, $-e^2/4\pi\epsilon_o r$ as the reference state.

It is possible to test the perturbation potential with data not only for the hydrogen atom, but also with hydrogen like single electron ions where $Z > 1$. The first order perturbation correction, δV, of a quantized energy level, δE_{nl} is the integral.

$$\delta E_{nl} = \int_0^\infty \phi_{nl}^* \phi_{nl} \delta V(r) r^2 dr \tag{21}$$

Where $E_{nl}^o \sim -Z^2/n^2$ with n in this relation being the principal quantum number (1,2, . .), l being the angular momentum quantum numbers (0,1,2, . .), which are the eigenvalues using a strict $1/r$ potential.

$$E_{nl} = E_{nl}^o + \delta E_{nl} + \sum \{ \, Second \; Order \; Terms \, \} \tag{22}$$

The integrals that are going to show the greatest values are those that have the maximum as $r \rightarrow 0$ which are the S states and in particular the *1S* state. Computation of the first order correction of the *1S* state is shown in Table II where $\gamma = 1$.

For an experimental reference one electron atoms and ions measured ionization energies will be used rather than dealing with complex problem of dynamic polarizability in a radiative transition that will affect the measured difference between energy levels.

$$E_{nl} = E_{nl}^o + \delta E_{nl} - \frac{1}{2}\alpha \mathbf{E}^2 + \sum \{ \, Second \; Order \; Terms \, \} \tag{23}$$

The dynamic polarization term that is schematically included in equation 23 as $-\alpha \mathbf{E}^2/2$ will try to cancel the first order corrections δE_{nl} in spectroscopic measurements but not in electron scattering experiments measuring ionization energies where the polarization is strongly suppressed.

TABLE 2. Matrix elements for hydrogen with the corrected potential for $\delta V(r) \sim \int_{\infty}^{r} u^{*} u dr' - 1/r. 10^{-21}$ *joules* $= .624 \times 10^{-3}$ *eV*

Matrix Element	***computed joules*** $\times 10^{-21}$
$< 1S \mid \delta V \mid 1S >$	1.137269
$< 2S \mid \delta V \mid 2S >$	.1421634
$< 3S \mid \delta V \mid 3S >$	.04212271
$< 4S \mid \delta V \mid 4S >$	.01777055
$< 2P \mid \delta V \mid 2P >$	-.000004361675
$< 3P \mid \delta V \mid 3P >$	-.000001399561
$< 4P \mid \delta V \mid 4P >$	-.0000006062336
$< 1S \mid \delta V \mid 2S >$	.4020936
$< 1S \mid \delta V \mid 2P >$	.001247904
$< 2S \mid \delta V \mid 2P >$	.0004432168
$< 2S \mid \delta V \mid 3S >$	.07738417
$< 2S \mid \delta V \mid 3P >$	.0002626220
$< 2S \mid \delta V \mid 3D >$	-.0000001834999
$< 2P \mid \delta V \mid 3S >$	.0002413868
$< 2P \mid \delta V \mid 3P >$	-.000002442469
$< 2P \mid \delta V \mid 3D >$	-.0000003872372

Atomic Ionization Energies

The largest correction is for the *1S* state that is on the order of 10^{-3} eV, which is a small correction, but easily measured. The question becomes how do you measure a property of a particle imposed from its self-reference frame and measured in the laboratory frame? In the laboratory frame the electron is taken as a point charge and it is accurately modeled for spectroscopic experiments for radiative transitions. A radiation field finds an electron through the electron's charge. The electron's charge is distributed over a compact volume with spherical symmetry so that the total charge can be taken as acting at its center of symmetry. Therefore, the point charge description can be used as a description as long as the electromagnetic field does not probe the core of the fermion. This probing is inhibited by the dynamic reaction of electrons to the field. A radiative interaction suppresses details of the structural characteristic of the fermion and its first order δE_{nl} interaction with the proton. The electron and proton make a simple two body problem and a radiation field would alter that quiescent state by polarizing the atom [16]. In order to detect the first order corrections due to the fermion structure a less invasive technique is required and that is to use another electron to scatter the bound electron out of the *1S* state and measure the ionization energy. A 14 eV electron is moving at $\sim .01c$ will have an interaction time of $\sim 10^{-17}$ seconds, which would be about 1% of the time required for the equivalent radiative transition. The incident electron will ionize the atom. This will lead to extracting δE_{10} as the difference between the computed 1S level of $-Ryberg \times \{reduced\ mass\ correction\}\ Z^2/n^2$ and the measured ionization energy as shown in Table 3

Ionization energy determination is an electrostatic scattering that triggers the removal of the final *1S* electron. Ionization is not a process that requires the generation of a photon to liberate an electron. The experimental data is collected by a radiation-less scattering process. There is no large scale dynamic electric field polarizing the atom prior to ionization except from the incident electron. The point that was made about electrostatic interaction being a contact interaction differentiates the ionization data from the spectroscopic data that requires and external dynamic field that will polarize the atom. Polarizability only becomes important when there is a distribution of both positive and negative charges spread over a volume. In Table 3 the first order correction computed for the *1S* state is slightly less than the measured ionization energy of hydrogen by $\sim$ 2%. This gives some support to the idea that the potential is not a singular 1/r potential in the vicinity of the electron's core.

TABLE 3. Comparison of the computed first order correction δE_{10} to the difference between hydrogen 1S state ionization energy and the standard 1S level. The effect of the additional neutron in deuterium reduces the measure first order correction to the 1S state significantly. Experimental data ref. [15].

Isotope	**Computed 1S** E^o_{10} $\times 10^{-21}$joules	**Measured Correction** $\times 10^{-21}$joules	**Computed** δE_{10} $\times 10^{-21}$joules	**exp./** δE_{10} **%**
^{1}**H**	2178.68640	**1.12447**	1.137269	99%
^{2}**H**	2178.68640	**.985**	1.137269	87%

Relativistic Effects

The first order energy change of the *1S* state dependent on, $\overline{T}$, the mean kinetic electron energy can be estimated by the Virial theorem from the energy of the state as $\overline{T} = .5|\overline{V}|$ where $\overline{V}$ is the depth of its potential well. This would push $\gamma \rightarrow 1.00001$ and yield a very small increase in the computed δE_{10} of $.0000403 \times 10^{-21}$ *joules*. These relativistic corrections for the electron are small until Z becomes much larger. As the electrostatic interaction is a contact interaction, there is no inhibition about interacting with a fraction charge components.

Deuterium

The measured correction of the ionization energy for deuterium is very different from the proton. The correction is reduced by 14% from the proton's value at $.985 \times 10^{-21}$ *joules*, which may indicate a very active three-body interaction between the proton-neutron-electron if the experimental data is accurate. This is different than a single proton and may indicate a contribution from the active short-range correlation data found for neutron-proton pair [17]. The short-range nuclear correlation activity is on a time scale much shorter than 10^{-17} seconds, which may contribute to the increased binding of the *1S* electron to the deuterium nucleus as compared to hydrogen. This increased binding cannot be attributed to a relativistic effect.

1S Corrections for $Z > 1$

The corrections to the *1S* state that is extracted through the ionization energy are interesting for two reasons. First it will be the largest correction to the energy eigenvalue because this state has the greatest overlap at the nucleus and secondly the nuclear charge can be varied by just going up the periodic table. Schematically the Schrödinger equation can be written in a charge dependent form as shown in equation 24, where Z is the net nuclear charge.

$$\widehat{\mathbf{H}}(Z)\psi_{1S}(Z) = E(Z)\psi_{1S}(Z) \tag{24}$$

The *1S* solution $\psi_{1S}(Z)$ can actually be made to represent a product of a set of single nucleon-charge interactions.

$$\psi_{1S}(Z) \sim e^{-Zr} = e^{-r} \times e^{-r} \times e^{-r} \ \ldots Z \ times \tag{25}$$

Where the lumped Z first order energy correction is:

$$\delta E(Z) = < \psi_{1S}(Z)|\delta V|\psi_{1S}(Z) > \tag{26}$$

This lumped solution assumes the nucleus is a single point charge, which is a poor assumption on the time scale that would affect δE_{10}. The solution can be altered to reflect the individual charge based couplings by factoring. This is closer to the physical reality required to understand the nuclear ground state which is coupled to a single $1S$ electron. The ground state dynamics of each nuclear charge must be accounted for in the electron's energy. Dynamic effects in and between the nucleons cannot be ignored because their contribution may not average to zero. The nucleus cannot be viewed as identical particles because their dynamics differentiates the individual interactions with the electron.

Writing out the matrix element for $Z > 1$ shows how the first order corrections end up as a sum of individual pair-wise electron-nucleon interactions rather than a bulk interaction with the entire nuclear charge. This differentiation is forced because all protons are no longer identical as they will be transforming because of their short-range correlations. The first order energy corrections is then a sum over the individual nuclear charges.

TABLE 4. Computed first order energy correction to the *1S* state due to the electron charge radius. For Z greater than 1 the correction to the *1S* state show a weakening bond where as the experimental data shows a strengthening of the bond. The Schrödinger equation solution treats a simple lumped potential only dependent up *Z*. Whereas, the binding appears to be a set of individual couplings to the charges of the separate nucleons. Experimental data from ref. [15].

Z	**elements**	***experimental difference*** joules10^{-21}	***computed*** $\delta E_{10}(Z)$ joules 10^{-21}
1	^{1}**H**	**1.12447**	1.137269
	^{2}**H**	**.985**	1.137269
2	^{4}**He**	**-4.2**	17.81
3	^{6}**Li** & ^{7}**Li**	**-3.5**	88.19
4	^{9}**Be**	**-12.1**	272.5
5	^{10}**B** & ^{11}**B**	**-12.5**	650.4

$$\overline{\delta V} = \frac{1}{Z}\sum_{i}^{Z}\delta V_i = \frac{1}{Z}\sum_{i}^{Z}(\frac{1}{r_i} - \int_{\infty}^{r_i} u^*(r_i')u(r_i')dr_i') \tag{27}$$

$$\delta E_{nl}(i) =< \phi_i|\delta V_i|\phi_i > \tag{28}$$

Where:

$$\delta E_{nl} \ =< \phi_1\phi_2 \ldots \phi_Z|\overline{\delta V}|\phi_1\phi_2 \ldots \phi_z >= \ \frac{1}{Z}\sum_{i}^{Z} < \phi_i|\delta V_i|\phi_i >= \ \frac{1}{Z}\sum_{i}^{Z}\delta E_{nl}(i) \tag{29}$$

This is a very different result than taking a single charge Z and computing a first order corrections. This sum will have terms that are much reduced from a lumped calculation. Experimentally the numbers are found in Table 4 and have more in common with the single proton corrections of equation 29 rather than treating the nucleus as a lumped *Z* charge found in column 4 of Table 4. The minimum features that must be added to the potential terms in the calculations are the nuclear charge distribution and the polarization of that charge.

There is a great deal of physics buried in trying to understand the deviations of the ionization energies from the *1S* state as a function of nuclear charge, particularly the negative value of the corrections as *Z* increases beyond 1.

Discussion

To test the deduction that an elementary fermion produces an electrostatic field, $u^*u \Rightarrow |\mathbf{E}|$ rather than a simple $1/r^2$ field imposes two constraints on time and space. For an electron's field the effects in space are on a scale of $\sim 10^{-13}$ meters or less. The constraint on time to measure the effect requires a time scale which does not disturb the bound system by polarizing the atom or ion. This eliminates using radiative transitions for examining a bound state such as the hydrogen atom. The ability of a quiescent bound electron state to be measurably polarized by nuclear charge dynamics appears to be easily detected and surprisingly large.

There are two different categories of nuclei being examined: the proton and all other nucleons which have more than one component. The ionization energy difference between hydrogen and deuterium is a major piece of data on the participation of the electron in transient nuclear dynamics. This difference is not small and considering the minuscule volumetric overlap, the change in ionization energy for deuterium is large. No simple corrections have been found to account for this shift that increased the binding.

The question of how to reduce $\delta E_{10}(Z)$ to increase the binding energy of the state looks to be a property of the nucleus. There is a small 1% difference of $2.432 \times 10^{-23} joules$ for the *1S* state of the hydrogen atom between the computed first order correction and the measured data. The effect for deuterium is much greater with a 18% reduction from experiment to computation. There is no change in the net charge between the two case but there is a change in the available electrostatic charge that can be polarized in the nucleus. For the three elements above helium the contributions become negative. It appears that during the short range correlation periods within the nucleus very strong local fields are generated that produces a polarization term affecting the *1S* electron: $-\alpha_n \mathbf{E}_n^2/2$, which is capable of dominating the first order correction δE_{10}. Where both $\mathbf{E}_n$ and α_n are features of the nuclear short range correlation process and α_n is proportional to the active nuclear volume. This effect becomes more predominant as the nucleon count increases.

We have tried to avoid the assumptions and problems of considering radiative transitions studied in quantum electrodynamics. The radiative transition is a much more complex process than the threshold ionization experiment. By introducing a different potential any future perturbation calculation will have to propagate this change through to computing the radiative corrections. Its is not easy to say what will result for the corrections, but they should be significant.

Appendix

Differential Equations in Self-Reference Frame

Starting in the self-reference frame with the conservation of energy relation:

$$E^2 = p^2c^2 \implies (\pm E)(\pm E) = (pc)(pc) \tag{30}$$

Factoring the two equations and replacing terms with their associated operators two differential equations result with a dependence in time and space.

$$\pm i\hbar c \nabla u(\mathbf{x}) = E u(\mathbf{x}) \tag{31}$$

$$\mp c\hbar k g(\tau) = \mp \hbar\omega g(\tau) = i\hbar \frac{\partial g(\tau)}{\partial \tau} \tag{32}$$

The scale of uncertainty in space, ϵ, enters the spatial equation as a random offset that is greater than zero. In the spatial differential equation becomes a second order differential equation.

$$u(\mathbf{x}) \rightarrow u(\mathbf{x} + \epsilon) \tag{33}$$

$$u(\mathbf{x} + \epsilon) = u(\mathbf{x}) + \epsilon u'(\mathbf{x}) \tag{34}$$

$$\nabla u(\mathbf{x} + \epsilon) = \nabla u(\mathbf{x}) + \epsilon \Delta u(\mathbf{x}) \tag{35}$$

$$\{\nabla u(\mathbf{x})\}_r \rightarrow \frac{\partial u(r)}{\partial r} = u'(r) \tag{36}$$

$$\{\Delta u(\mathbf{x})\}_r = \frac{\partial^2 u(r)}{\partial r^2} + \frac{n-1}{r}\frac{\partial u(r)}{\partial r} = u''(r) + \frac{n-1}{r}u'(r) \tag{37}$$

$$u''(r) + (\frac{n-1}{r} + \kappa\{1 - i\gamma\})u'(r) - i\kappa^2\gamma u(r) = 0 \tag{38}$$

Similarly in the first order time dependent equation the uncertainty in time enters as $\Delta\tau = \epsilon/c$.

$$g(\tau + \Delta\tau) = g(\tau) + \Delta\tau \frac{\partial g(\tau)}{\partial \tau} + \ . . \tag{39}$$

$$\frac{\partial g(\tau + \Delta\tau)}{\partial \tau} = \frac{\partial g(\tau)}{\partial \tau} + \Delta\tau \frac{\partial^2 g(\tau)}{\partial \tau^2} + \ . . \tag{40}$$

$$\pm i\omega(g(\tau) + \Delta\tau \frac{\partial g(\tau)}{\partial \tau}) = \frac{\partial g(\tau)}{\partial \tau} + \Delta\tau \frac{\partial^2 g(\tau)}{\partial \tau^2} \tag{41}$$

$$\frac{\partial^2 g}{\partial \tau^2} + (\omega_c \mp i\omega)\frac{\partial g}{\partial \tau} \mp i\omega\omega_c g = 0 \tag{42}$$

Numerical Integration

The properties of ${}_1F_1[a, b, r]$ can be expressed in the following terms:

$${}_1F_1[a, b, z] = 1 + \frac{az}{b} + \frac{(a)_2 z^2}{(b)_2 2!} \ldots . + \frac{(a)_n z^n}{(b)_n n!} + \ \ldots \tag{43}$$

where $(a)_o = 1$ and :

$$(a)_n = a(a + 1)(a + 2) \ldots (a + n - 1) \tag{44}$$

In the case of three dimensions when *b=2* and $a = 1 - i\gamma$ with $z = (1 + i\gamma)\kappa r$ the series can be simplified.

$$e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ \ldots . \tag{45}$$

$${}_1F_1[1 - i\gamma, 2, z] = 1 + e^{-iatan(\gamma)} z + e^{-i(atan(\gamma)+atan(\gamma/2))} \frac{z^2}{2!} + \ldots . . \tag{46}$$

Where the series has form of the series for e^z with each n term being multiplied by g_n

$$g_n = e^{-i\sum_{i=1}^{n} atan(\frac{\gamma}{n})} \tag{47}$$

When $\gamma \to 1$ the result is just a rotated version of the e^z series. The series for ${}_1F_1$ looks like it can be treated as inner products of two infinitely long vectors where the modulation of the base vector represented by the series for e^z is multiplied by a vector generated from g_n.

Care has to be taken with ${}_1F_1$ function in numerical calculation [18]. The series expansions for small and large arguments of the ${}_1F_1$ functions from ref. [14] were computed to a minimum of 12 place precision. The transition point $|z| = 25\sqrt{2}$ was selected between the positive and negative power series in z. The Γ functions were expanded in a power series of eight terms using the poly-Γ functions at the fixed point $1 + i$. The potential differences are computed by integration in two stages from $\epsilon\times\ 5\times10^7$ taken as infinity to $\epsilon\times\ 10^4$ then continuing to a point below $\epsilon\times\ 10^4$ where the value is required. The upper integral is done in 5×10^7 steps. The integrals for the matrix elements are performed from 0 to $\epsilon\times\ 10^4$ in 5×10^7 steps. The bulk of the contributions to the matrix elements are at radial distances less than 10ϵ where the potential deviates most from Coulomb's law.

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History

After completing this paper it was realized this is not the first time this particular problem was seen. At the end of the 1966 academic year JPW started working in the Columbia Radiation Laboratory for a group run by P. Kusch. At this time I.I. Rabi was teaching his last course and retiring. C. Towens was moving to Berkeley and in Towen's laboratory room P. Kusch and P. Cahill were changing course from the normal spectroscopic measurement and beam experiments that P. Kusch was involved with since 1938 to start doing ionization and dissociation measurements. These were techniques he had learned as a research assistant at the University of Minnesota in the years 1935-1937. The precision radiative transition measurement to determine the anomalous magnetic moment of the electron and other deviations in transition spectra were at that time only analyzed by using the tools of quantum electrodynamics and Prof. Kusch was uncomfortable with this type of analysis. His preferred analysis technique which used a closed form solutions with the minimum of assumptions that could be experimentally constructed and then the results could be compared. Unfortunately, political events intervened. The student riots in the spring of 1968 started a series of events that led P. Kusch into the administration of the university. A number of key experimental physicists also departed at this time. Then after a few years of administrative duties he moved to Texas and the ionization research was never restarted.

I suspect that P. Kusch was well aware of the unexplained offset in the ionization energy for 1S hydrogen and this large discrepancy was not the subject of much theoretical interest. However, as a student after building components for the low energy electron gun to be used for the ionization work I was urged to try a standard quantum mechanical analysis on the ionization problem because it should yield some interesting results without having to resort to the use of higher order perturbation analysis required for quantum electrodynamics. From the variety of their previous work with the lithium isotopes one of their aims was to gain more information about the nucleus and its active potentials. A cleaner and simply experimental and theoretical technique were what they were looking for in obtaining data about the nucleus. The path to this end had go back even further in time to 1932-34 where quantum mechanics failed to incorporate a general relativistic basis so the electron's potential could be generated. In both instances political forces swamped the technical efforts.

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