Statistical Independence in Quantum Mechanics

John P. Wallace

Casting Analysis Corp.,
8379 Ursa Lane, Weyers Cave,
Virginia 24486 USA [*]

Michael J. Wallace

Phoenix, AZ 85050 USA

Statistical independence when applied to spaces and not just variables become a powerful tool for encapsulating quantum objects like fields and particles. It has been used on particle descriptions to generate base properties. It also can be used on massless fields. The solar neutrino survival data correlates well with an analysis for the electron neutrino, ν_e being massless. The ν_e remains a massless charge free field as proposed by Pauli. The ν_e wave function generates a spatial density oscillation that reduces its scattering potential by one-half. The analysis produces a confirmation of the original quantum conjecture by Planck and Einstein that radiation is quantized and can be explicitly verified from the allowed solutions to the field state equations of their statistically independent embedded space. This result exposes one of the major defects in the foundation of quantum mechanics, the missing mechanism that quantizes a massless field and generates inertia for particles. *draft 4 August 2019*

CONTENTS

I. INTRODUCTION

The foundation of quantum mechanics suffers a number of defects. Two defects in the subject are the missing mechanism that quantizes the massless fields and generation of inertia or the self-energy of massive particles. What is not explained by quantum mechanics are the origins of material properties measured with limited precision. Key to understanding how properties are generated is tied to the concept of statistical independence not of variables but of entire spaces.

Physical statistical independence investigated here is a completely quantum concept and not connected to classical Brownian motion, the many particle processes of statistical mechanics, or quantized chaotic orbit theories (Gutzwiller, 1990). Our introduction to the concept came through Mark Kac, who continued a very productive line of research after studying diffusion and Brownian motion (Kac, 1947) (Kac, 1949). In the late 1940s his work was subsequently used in a quantum description of nonrelativistic path integral analysis that looked much like diffusion. Kac's work importance to us was a need to understand the quantum diffusion of hydrogen and its isotopes in metals, particularly iron. This application can be treated as a combination of classical diffusion with quantum potential wells. In high density metals where the proton is treated as unscreened in shallow potential well whose bound state eigenvalues are determined by the mass of the isotopes (Wallace, 2011). Then the question arose what is minimum potential well depth supporting an eigen state? The answer yields a very curious result. This minimum bound state for a vanishingly small potential well for a particle produces the Compton relation independent of the potential wells depth and only dependent on the potential well's radius (Wallace and Wallace, 2011). This is a limiting case application of the Schrödinger equation that begins to expose the source of a inertial mass.

In the middle of the 1960s, Kac posed another interesting problem relevant to those trying to understand high energy scattering data to work out the structure of nucleons (Kac, 1966): *Can one hear the shape of a drum with holes?* If you beat on an arbitrary drum will its audible spectrum contain enough data to accurately reconstruct the drum? In the 1950s Kac's mathematical

[*] jpw@castinganalysis.com

interests were applied to discrete random processes and the concept of statistical independence of random variables (Kac, 1959). Kac realized that part of the statistical nature of quantum mechanics involves the less than well defined nature of the quantum particle. The symbolism of an exicted drum head might strike his physics colleagues as something familiar. There is a basic question that underlies both subjects and that is the precise definition of information; types of information and its origin.

The random walk generated by Brownian motion was a nice analogy but not physically realistic for fleshing out the statistical basis of quantum mechanics. The path integral approach that Kac was exploring with Feynman was an attack on the dynamic problem of quantum objects; unfortunately, their elementary particles entered in as Newtonian point masses and point charges. The fields entered in as plane or spherical waves. These assumptions remove consideration of the particle/field structure. Kac's consideration of a defective drum made the particle a real structure a point to be considered. The scale of the drum head removes the point particle assumption as a barrier to having a particle with a defined scale. The complex spectra of the drum was a start at producing a basis for a scattering particle's properties and immediately brought into question the information content of the spectra.

Kac, with his drum analogy, was able to take the problem to a lower level and in a different direction by showing that the inversion techniques operating on the measured spectral output fails to determine the true structure of the drum because that requires more data than can ever be collected. Kac's failed inversion analysis highlights the high energy potential scattering problem, that of determining an unknown potential's structure by scattering experiments such as done with accelerators.

A. The Lowly Potential

The legacy opposition to exploring the structure of elementary particles and fields came from the theorist that were developing quantum electrodynamics and the effective field theories for high energy. The reason for this was simple, they embraced a method to rid themselves of having to explain the electrostatic potential and other force potentials. This was accomplished by assuming there was an exchange boson to explain each and every force: electromagnetic, gravity, strong and the weak force. This was a terrible mistake and its origin was in the failed attempts at explaining the origin of inertia and inertial mass. The point mass and point charge of the 18th, 19th, and 20th century was a useful tool and to abandon it meant also abandoning the conveniences of the mathematical continuum.

In order to have a potential independent of an exchange boson it was necessary for the particle to have a structure, beyond that of point mass. Kac's drum head with holes explains nicely the failure of high energy experiments to yield elementary particle structure information. The canard that high energy accelerator experiments are really a high resolution microscope of structure was exposed and ignored. In order to experimentally crack the problem, quantum mechanics itself provides the assistance. Quantum structures scale from elementary particle to collections of particles behaving with simple quantum properties as superconducting currents and magnetic excitation. These larger structure are more easily explored.

B. Exposing the Self-Reference Frame

Instead of the acoustical problem that Kac posed, the problem was recast into one of a near-field electromagnetic scattering problem using multiple simultaneous frequencies to examine an unknown object's: range, scale, conductivity and magnetic permeability (Siegfried, 1983). Maxwell's equations can be solved explicitly for the forward problem, and checked against known materials (Dodd and Deeds, 1968). It is found that when unknown materials are used their conductivity and permeability properties can be accurately extracted if they behave with the following two material constitutive restrictions.

$$\begin{aligned} \mathbf{B} &= \mu \mathbf{H} \\ \mathbf{J} &= \sigma \mathbf{E} \end{aligned} \tag{1}$$

From the solutions of the forward problem the domain of allowed solutions can be plotted out and the allowed reflection responses are defined by the restrictions found in Eq. 1 (Wallace, 2011). This is no different than a photon mediated high energy scattering problem. As long as these conditions are in place a rough inverse problem can be solved to the precision of the measurements and produce useful information. The inverse analysis, analogous to Kac's drum problem, fails rather spectacularly if the material is capable of absorbing the incoming energy and processing it into an excited quantum state on the scale of the object being examined. The object then becomes Kac's drum head, where the holes in the drum form a spectrum of their own.

For the transvese electromagnetic problem the Faraday-Maxwell description now needs to be extended, but there is not much in the scattering data to tell one how to accomplish this task when done in well annealed iron or a low carbon steel. To measure the fields properties an experiment must be done to capture the dispersion relations of the newly observed fields. The new data acquired resolved a longitudinal field with an effective mass very different from the original massless transverse fields (Wallace, 2009a) (Wallace, 2009b). The dispersion relation for a well annealed iron or a low carbon steel is actually more complex producing three fields with the most interesting, a propagating field, with a mass of 10^{-9} of an electrons mass. This very light spin wave has a correspondingly large scale $\sim .14$ meters. A low frequency magnetic field drives the creation of exciton that is an oscillating longitudinal magnetization wave with mass and large scale structure. This solved a major experimental problem because now a quantum particle structure can be examined in detail on a lab bench. It turns out the structure of this particle at relativistic energies matches the behavior of a spin zero boson and not a fermion. This light boson can collect in large numbers because of its low mass. A Bose-Einstein condensate can form at temperatures that exceed the Curie point of the metal making it even easier to study. This light exciton must be treated from the beginning as a relativistic particle because of the very small mass (Wallace and Wallace, 2014).

II. ENERGY CONSERVATION

To discover how inertia is produced for this magnetic excitation energy conservation must be examined. Quantum mechanics and relativity were discovered almost together in 1900-1905. They were treated as separate subjects, because the connections between the two was unknown. Both fields were forced into a mold that mirrored classical mechanics and this further separated the fields making both subjects even more opaque. It was not until extensive experimental measurements were made that two conservation of energy laws emerged, one for a free massless field and one for a free massive particle both in a potential free region.

$$E = \hbar\omega$$
$$E^2 = (pc)^2 + (mc^2)^2 \tag{2}$$

These relations were not derived from fundamentals of about space and time. What was derived by Einstein was an equivalence between self-energy mc^2 associated with mass and kinetic energy. In his book *Relativity* the second expression does not appear because it could not be derived physically and it was latter produced by a mathematical convention (Einstein and et. al., 1952). Classical non-relativistic quantum mechanics approximates two problem well: the hydrogen atom and the harmonic oscillator. It does not accurately handle the free particle, along with diffraction, refraction, pair-production, nor generate the general description of quantum particles: boson and fermion. This collection of defects reflect a poor understanding the two energy conservation relations and their origins in relativity. There is a myth that quantum electrodynamics, a method of calculation, has made quantum mechanics the most accurate theory ever. Quantum electrodynamic's non-unique set of corrections are considered even by R. Feynman, one of the originators, not a description of physics, but only a method of calculation. The non-uniqueness allows result to use non-physical properties (potentials with singularities) to generate any desired number. These two conservation of energy relations allows one to derive a description of the space where particle and field properties are generated, the self-reference frame and their behavior in the laboratory frame. This is a two part derivation to generate both structure and dynamics in two separate spaces. These spaces are not in hierarchy, but complimentary where each space's existence is dependent on the other.

1. Self-Reference

Starting with the massless field conservation of energy and randomizing motion for that field begins the derivation to produce structure and inertia. This randomizing process is generated independently when the dynamics equation is derived in the next section. The particle's structural form in the space referenced on the particle itself can be generated by Taylor expanding the momentum and energy operators around r and τ of the field with the random spatial parameter, ϵ and time parameter c/ϵ resulting in two differential equation, one for the spatial structure, $u(r)$, and one for the time dependence, $g(\tau)$, which are derived in detail (Wallace and Wallace, 2014) (Wallace and Wallace, 2015). The derivation begins with the expansions:

$$E = pc \quad \rightarrow \quad E = p(r + \delta r) \quad \rightarrow \quad E = p(r + \epsilon)c$$
$$E = \hbar\omega \quad \rightarrow \quad E(\tau + \delta\tau) = \hbar\omega \quad \rightarrow \quad E(\tau + \frac{\epsilon}{c}) = \hbar\omega \tag{3}$$

The entire concept of a point mass and point charge with their associated infinite energies vanish in this picture along with the cut offs necessary in quantum electrodynamics. What also vanishes is the single virtual photon, which cannot be supported because it will change the information content of the laboratory frame.

Starting with the dispersion relation for a massless field in laboratory frame (Wallace and Wallace, 2015):

$$E = pc \tag{4}$$

The spatial dependent equation will be derived first where $u(\mathbf{x})$ is the spatial dependent function.

$$i\hbar c\nabla u(\mathbf{x}) = Eu(\mathbf{x}) \tag{5}$$

The scale of uncertainty in space, ϵ, enters the spatial equation as a random offset that is greater than zero. In the spatial differential equation becomes a second order differential equation.

$$\begin{aligned} u(\mathbf{x}) &\to u(\mathbf{x}+\epsilon) \\ u(\mathbf{x}+\epsilon) &= u(\mathbf{x}) + \epsilon u'(\mathbf{x}) \\ \nabla u(\mathbf{x}+\epsilon) &= \nabla u(\mathbf{x}) + \epsilon \Delta u(\mathbf{x}) \end{aligned} \tag{6}$$

$$\{\nabla u(\mathbf{x})\}_r \to \frac{\partial u(r)}{\partial r} = u'(r) \tag{7}$$

$$\{\Delta u(\mathbf{x})\}_r = \frac{\partial^2 u(r)}{\partial r^2} + \frac{n-1}{r}\frac{\partial u(r)}{\partial r} = u''(r) + \frac{n-1}{r}u'(r) \tag{8}$$

The result of expanding the differential forms of the dispersion relation with the disorder parameters is a pair of differential equation for the spatial variable, r, the radial coordinate and the time coordinate, τ. The angular coordinates in spherical geometry are lost in the random behavior introduced to generate a particle description located on the instantaneous center of symmetry of the particle. The field equation are written in terms of the dimension of space, n, with the parameters $\gamma = E/mc^2$, $\omega_c = mc^2/\hbar$, and $\kappa = 1/\epsilon$. The resulting differential equation from expanding the conservation of energy relation and referenced to the particles instantaneous center of symmetry.

$$\frac{\partial^2 u(r)}{\partial r^2} + (\frac{n-1}{r} + \kappa\{1 - i\gamma\})\frac{\partial u(r)}{\partial r} - i\kappa^2\gamma u(r) = 0 \tag{9}$$

The time dependent equation can also be expanded from the the dispersion relation $E = \hbar\omega$.

$$\frac{\partial^2 g(\tau)}{\partial \tau^2} + (\omega_c + i\omega)\frac{\partial g(\tau)}{\partial \tau} + i\omega_c\omega g(\tau) = 0 \tag{10}$$

The second order spatial equation has two solutions include the hypergeometric functions ${}_1F_1$ and U where A and B are constants (Slater, 1968):

$$u(r)_{fermion} = Ae^{-\kappa r}{}_1F_1[\frac{n-1}{1+i\gamma}, n-1, (1+i\gamma)\kappa r] \tag{11}$$

$$u(r)_{boson} = Be^{-\kappa r}U[\frac{n-1}{1+i\gamma}, n-1, (1+i\gamma)\kappa r] \tag{12}$$

What was discovered on inspecting these two solutions were properties consistent with the two families of particles with mass: boson and fermion (Wallace and Wallace, 2014). The first solution represents a fermion and the second solution represents a boson both with a real mass. All densities determined from the solutions retain spherical group symmetry, *U(1)*, so that charge can be extracted. The function $u^*(r)u(r)$ is used to define the particle's static electric field if it can support a charge (Wallace and Wallace, 2015). Charge properties can be determined by an analysis of the derivative $\partial\gamma/\partial\theta$, which will define quantized charge, mass independence of charge, and the dimensional dependence of charge where θ is the argument of the complex solution written as: $ArcTan\{Im[u(r)]/Re[u(r)]\}$. If there is no θ dependence in $u(r)$ the particle has a zero charge and cannot support an electromagnetic transition.

This space, the self-reference frame, is a primitive domain where no form of momentum is defined and the dynamics only refer to the relative stability of the particles. The equations are compatible with relativity through γ, which describes their behavior with different relative observers. Linear momentum, angular momentum and the magnetic moments are dynamic properties of the laboratory frame and are not part of the particle's information developed in the self-reference frame.

The importance of the self-reference frame is that as a statistically independent space it can generate the particle's self-energy. Independence means there is no mapping between the two frames, either in space or time. This independence is reflected in the Pythagorean sum required for the two components in the conservation of energy relation, Eq. 2, which adds the quadratic of the kinetic energy to the quadratic of the self-energy. Rather than add physical dimensions to the 3+1 space of the laboratory frame for additional particles it is possible for any particle or collection of related particles to establish an embedded private space statistically independent from the laboratory frame. This was first attempted by Dirac in 1932 (Dirac, 1932) with an introduced private time and ran into severe opposition from Pauli and Wessikopf (Pauli and Weisskopf, 1934) who used a counter argument that involved the same unfortunate Klein-Gordon equation.

2. Laboratory Frame Equation

The second half of the derivation requires generating the compatible dynamics in the laboratory frame. This also produces the mechanism that generates the basic statistical properties of quantum mechanics. To do this the concept of a potential is necessary and now it is based on the structure of the particle itself as derived in the self-reference frame. Within the relativistic conservation relation the potential is derived from the mass of the particle. The variation $m - m_o = \delta m$ represents the source of the potential interaction.

$$E^2 = p^2c^2 + (m_o + \delta m)^2 c^4 \tag{13}$$

$$E^2 - (m_o c^2)^2 = p^2c^2 + (2\delta m m_o + \delta m^2)c^4 \tag{14}$$

δm^2 is small relative to $2\delta m m_o$ and is dropped. The potential is taken to be $V = \delta mc^2$

$$E^2 - (m_o c^2)^2 = p^2c^2 + 2Vm_o c^2 + V^2 \tag{15}$$

$$\frac{E^2 - (m_o c^2)^2}{2m_o c^2} = \frac{p^2}{2m_o} + V(1 + \frac{V}{2m_o c^2}) \tag{16}$$

It is simple to derive something functional to replace the Klein-Gordon equation that conserves energy and compatible with relativity as a second order wave equation in the laboratory frame (Wallace and Wallace, 2014) (Wallace and Wallace, 2017). The energy operator, which is a first order time derivative, is taken as the total energy less the self-energy.

$$i\hbar\frac{\partial}{\partial t} \rightarrow E - mc^2 \tag{17}$$

Using the momentum operator and the correct energy operator equation 2 is converted into the resulting differential equation, which has two additional terms absent from the Schrödinger equation. The second order time dependent term embedded the propagating field equation more commonly found from electromagnetic theory of Maxwell. The second addition is a quadratic term in the potential, whose presence brings in the mechanics of the virtual field and pair-production naturally that is no longer an ad hoc postulate (Wallace and Wallace, 2017).

$$\frac{\hbar^2}{2m}\{\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}\} + i\hbar\frac{\partial\Phi}{\partial t} = (V + \frac{V^2}{2mc^2})\Phi \tag{18}$$

The above equation can be reduced to the standard Schrödinger equation for some bound state and free propagation problems. This comes at a cost of losing its compatibility with relativity. That reduction introduces errors that have been commonly corrected by perturbation techniques.

A. $V + \frac{V^2}{2mc^2} = 0$

The laboratory frame description of a massive particle yields the mechanisms of random behavior necessary to produce inertia, in the quadratic term of the potential. When the potential contribution in free space with no external potentials there remain two solutions to the above equation $V = 0$ and $V = -2mc^2$ where both solutions are equally weighted. The second solution represent a pair production allowed by the Heisenberg relation of particle and anti-particle the same as the original being described. The annihilation with either the original or the generated particle produces the statistical basis of quantum mechanics.

In the late 1920s matrix mechanics, Schrödinger equation and the Dirac equation were all essentially written down. They were not fundamentally derived from any understanding of the quantum processes or in the case of the Dirac equation forced to be a linear approximation. The problem they all suffered from was they did not include the correct relativistic basis. There is no such thing as a correct non-relativistic quantum description, at best it is an approximation that bars any understanding of structure of particles and fields.

B. Inertia

What is required to generate a mass from a primitive field are obstacles to aid in localizing a field moving at the speed of light. A set of obstacles that conserve energy in the laboratory frame are composed of field-anti-field pairs. Sometime the original field makes it through and other times it annihilates and its opposite number takes over being the propagating field. This results in a random displacement. If this process is truly random then the original field will be localized under some very specific conditions. Our original field's a self energy is taken as $\hbar\omega$ as well as for our final field as energy is conserved. To compute the rate of pair production the self-energy of the new pair becomes $2\hbar\omega$ with a mass equivalent equal to $2mc^2$. The localization is initiated in the laboratory frame so that the rate, R, of the pair-production can be computed from the Heisenberg uncertainty relation for energy.

$$R = \frac{1}{\delta t} \leq \frac{4mc^2}{\hbar} \tag{19}$$

At any time our field has a 50% chance of encountering a pair and compounding that a 50% chance of annihilating and passing the baton to the newly minted field. This equal weighting can be explicitly derived, see chapter 3 in (Wallace and Wallace, 2017). So in total it has a 25% chance of being replaced. This rate turns into an equality since the only virtual field pairs that can interact with original field must have the identical energy as these are conservative processes. This rate of replacement is $1/4$ the rate of pair production.

$$\frac{R}{4} = \frac{mc^2}{\hbar} \tag{20}$$

The inverses of the rate $R/4$ is a mean interval any particular field lives and the distance light can travel in that interval is ϵ which now can be computed from Eq. 20.

$$\epsilon = \frac{\hbar}{mc} \tag{21}$$

This is the Compton relation produced from a real disorder paramter, ϵ. The net effect on our field is set by the mean rate of exchanging fields and generating a locality for a particle with inertia as its local position is unknown to a mean random value ϵ. The angular coordinate description is lost in the self-reference frame as it is reset to the present position of the particle's center of symmetry. By randomizing the local location of the fields center of symmetry a particle is created with a finite scale along with local isotropy. This can be extended as the field is defined over all space along with the isotropy. The origin of the field always has to keep shifting after each annihilation to the replacement field's partner. This random-annihilation walk generates a location, a fuzzy location, but a location that can be described. The coordinates in time and space are now statistically independent of the original laboratory frame from where they were created. So from the laboratory frame with the physical property that allows pair-production for short intervals a localized entity can be created from something very rare an absolutely fair game of chance. This game of chance generates a statistically isolated space independent of the laboratory frame with the particle's instantaneous frame of reference tied to the current field. What is defined in this embedded space is the self-energy of the particle and this frame we call the self-reference frame.

III. QUANTIZATION OF FIELDS: COMPLEX DISPLACEMENT

Since mass is inversely related to the random variable ϵ to make mass complex ϵ must be made complex. By making ϵ complex it is equivalent to introducing a phase shift and this should be retarded so the transformation that will be used is found in Eq. 22 because $\epsilon > 0$ for generating a real mass. This random displacement is always positive in a spherical coordinate system as it is referenced from the instantaneous center of symmetry that is changing. Therefore for the complex displacement the relation in Eq. 22 is used.

$$\epsilon \rightarrow -i\epsilon \tag{22}$$

To transform the remaining parameters into field equations to test the freshman's conjecture about a complex mass, it is first necessary to understand how γ in the self-reference frame transforms.

$$\epsilon \rightarrow -i\epsilon \quad then \quad \gamma = \frac{E}{mc^2} = \frac{\hbar\omega_c}{\frac{\hbar}{-i\epsilon c}c^2} = -i\frac{\epsilon}{\epsilon} = -i \tag{23}$$

$$\epsilon \rightarrow -i\epsilon \quad then \quad \omega_c \rightarrow i\omega_c \tag{24}$$

For the case in the self-reference frame when the Compton wave length is set equal to the random displacement parameter, $-i\epsilon$, then $\gamma \rightarrow -i$. This is one of the more important relationships derived, because it essentially enforces the quantized condition on the resultant field. In particular this is also the quantum condition for the photon energy. A particle in the self-reference frame to participate in an electromagnetic transition or the exchange of energy with an electrostatic field must be able to change γ. For a massless field either boson or fermion it is necessary that γ is a fixed complex constant that cannot vary. Therefore, the field either exists or doesn't exist with no decay mechanism. The constraint that $\gamma = -i$ confirms the original conjectures by Planck and Einstein that radiation is quantized. This is not the mechanism of energy exchange for an electrostatic interaction only for a radiative transition where a real photon is exchanged.

The self-reference frame places a strict conditions on the material parameters that are defined in this independent space. Mass does not appear as a free parameter in the self-reference frame because it is a description of fields. If the equivalent complex random displacement is applied to the particle description $\epsilon \rightarrow -i\epsilon$, $\kappa \rightarrow i\kappa$ and $\gamma \rightarrow -i$. The new differential equations become:

$$\frac{\partial^2 u(r)}{\partial r^2} + (\frac{n-1}{r})\frac{\partial u(r)}{\partial r} + \kappa^2\gamma u(r) = 0 \tag{25}$$

$$\frac{\partial^2 g(\tau)}{\partial \tau^2} + \omega^2 g(\tau) = 0 \tag{26}$$

The solutions in three dimensions can be directly written.

$$u_{boson} = A\ e^{-i\kappa r}\ U[1, 2, 2i\kappa r] \tag{27}$$

$$u_{fermion}(r) = B\ e^{-i\kappa r}\ {}_1F_1[1, 2, 2i\kappa r] \tag{28}$$

$$g(\tau) = Ae^{-i\omega\tau} \tag{29}$$

The complete solutions are then:

$$\phi(r,\tau)_{boson} = A\ e^{-i(\kappa r - \omega\tau)}\ U[1,2,2i\kappa r] \tag{30}$$

$$\phi(r,\tau)_{fermion}(r) = B\ e^{-i(\kappa r - \omega\tau)}\ {}_1F_1[1,2,2i\kappa r] \tag{31}$$

Now that both elementary particle and field structures have been derived their density functions in three dimensions can be plotted in Figure 1. The total wave function in the self-reference frame $\phi(r,\tau) = u(r)g(\tau)$ the time dependence being of the form $e^{-i\omega\tau}$ becomes a constant factor in the probability density function. The particle density in the self-reference frame in three dimensions is given by the expression $u^*(r)u(r)r^2$. The core of density $u^*(r)u(r)$ in the case of a massive fermion is proportional to the static electric field and removes the $1/r^2$ singularity of the point electron at its center of symmetry (Wallace and Wallace, 2015). In the case of the massive boson the properties of weak charge result and the description is found in (Wallace and Wallace, 2014). For the massless fields the boson density is a constant as it is for the photon field. However, for the fermion field it has a spatial oscillatory behavior, that will affect a number of properties. It is the energy dependent oscillatory character of the density function that is of primary interest as it reduces the particles interaction cross section.

$$\begin{aligned} < u^*_{photon} u_{photon} r^2 > &= 1 \\ < u^*_{neutrino}(r) u_{neutrino}(r) r^2 > = < Sin^2 \kappa r > &= \frac{1}{2} \end{aligned} \tag{32}$$

The mean value of the Sin^2 term is exactly one half. This behavior in the spatial portion of the wave function is unique among particles and will lead to a reduction in detected sensitivity by exactly 50% in measured data whether from solar or reactor generated electron neutrinos.

The extensive literature on the neutrino-cross section as a function of energy that result are dynamic calculations at a level above of the density calculation for the neutrino in the self-reference frame (Formaggio and Zeller, 2012). The kinematic models do not involve the structure of the particles themselves, only their bulk properties and allowed interactions. It is not necessary to involve the specific mechanisms for the energy dependent calculation of cross-sections, because the correction being introduced will affect the neutrino across its entire energy range uniformly.

The original question about a complex mass first generates two massless fields that appear to have real physical counter parts: photon and the electron neutrino. First is a boson with a unit density characteristic of a basic photon and then a massless elementary fermion representing a neutrino. These are solutions in the self-reference frame and not in the laboratory frame where their complete structure is developed. Both solutions are of massless fields showing no perfered local structure. This was forced by $\gamma = -i$ being fixed complex constant. Any other values of γ produced divergent solutions that are not valid. Divergence here means that the density functions grow larger with increasing r, which is neither the property of a physical realizable particle or field. Fixing γ for massless field also insures the independence of the speed of light in any reference frame. This restriction on γ is a requirement for the quantization of the field for both the photon and neutrino. The detailed behavior as a function of γ are also found for their associated anti-particles (Wallace and Wallace, 2014). The fermion field had zero charge because the wave function, *u(r)*, has no variation in its complex argument.

In the self-reference frame the harmonic time dependence of a stable entity that starts with a private time dependence when the frame is created with no previous history. All entities whether a particle or a field come with their own clocks, via their time dependence, and are essentially isolated by the statistical independence of the space in which they were generated. The only exception is when two or more particles share the same clock either from being created at the same instance or interacting with one of two fields or particles that were created as a pair. This behavior is important for understanding entanglement. One requirement for relativity is a measurement scale and a time base and these conditions are satisfied for each individual particle and quantized field.

IV. BOREXINO DATA

The ν_e detection method is to monitor the scattering process (ν_e, e^-) in a large liquid scintillator (Derbin and group, 2016). The Borexino analysis assumes the standard solar model chain of coupled fusion reaction and decays generates a distribution of isotopes that is accurate from an end point analysis of star surface chemistry. To do this analysis one has to assume that knowledge of all possible reactions are included and accurately accounted for including the dependence on the distribution of material through out the sun as a function of depth and temperature. The second assumption is that the calculated kinetic neutrino cross section is assumed to be correct, because there is a good understanding of the weak processes. The strength of the analysis and experiment is that most of the activity with the ν_e occurs for processes that can be individually isolated. The data of

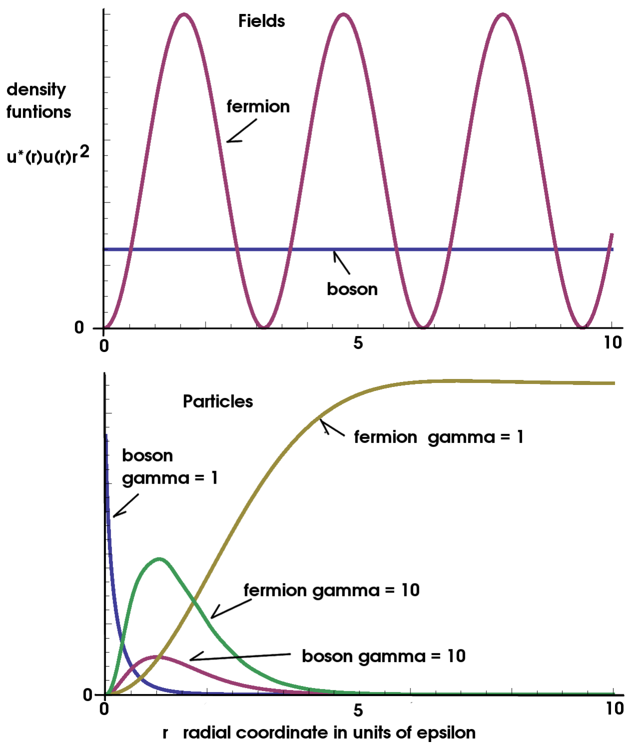

FIG. 1 **Density functions of fields and particles in the self-reference frame. The individual density scales are arbitrary so the functions separate. The parity problem of the massive bosons can be seen at $r = 0$ as the density function in its dependence on γ and is independent of γ for fermions.**

interested is presented in Table I.

TABLE I **Probability of solar neutrino survival data from Borexino (Derbin and group, 2016). The *pp* and the ^8Be have continuous neutrino spectra down to zero energy. The *pp* process is the dominant process but with a small detection cross-section making it more difficult to resolve at low energy. The mean on the unweighted sum of the survival probability is .49 ± .11**

Energy	Process	Mean	Low	High
.3 - .4 MeV	$^1H + {}^1H \rightarrow {}^2H + e^+ + \nu_e$	.64	.52	.76
.89 MeV	$^1H + {}^1H + e^- \rightarrow {}^2H + \nu_e$	.62	.47	.79
1.5 MeV	$^7Be + e^- \rightarrow {}^7Li + \nu_e$	.52	.46	.58
3 - 18 MeV	$^8B + e^- \rightarrow {}^8Li + \nu_e$	.38	.27	.51
5 - 18 MeV	$^8B \rightarrow {}^8Li + e^+ + \nu_e$	.31	.22	.43
.3 - 18 MeV	Means (theory .5)	.494	.388	.614

If the neutrino density function from Eq. 32 is correct there will be a factor multiplying the flux measurements which is $.5$ and the average of the unweighted five process is $.49 \pm .11$. This unweighed result is very close to the expected decrease computed for the reduction produced by the neutrino density function. Because of the manner of experimentally isolating individual components the best comparison that can be made is an unweighted mean. Tying the deficit of neutrinos to a reduction in neutrino flux rather than a reduction in detector sensitivity across a significant energy range leads to a false conclusion about ν_e mass. The data indicates the solar ν_e is most likely a stable massless field.

V. DISCUSSION

The structures for a massless field opens some new avenues for understanding the complex data now being produced by the large neutrino detectors. With the electron neutrino being massless like the photon its experimental properties have to also encompass refraction effects mediated by the weak force transition allowed in the matter it is traversing.

The measurement problem of quantum mechanics is now open to different solution as there are two frames of reference mutually connected without the the necessity of an observer. The combination of lower dimensional components to construct baryons and possibly more of the high energy particle an fields looks promising. The mathematics of spaces not dependent on the mathematical continuum will change the tools used.

ACKNOWLEDGMENTS

First to Jack Steinberger who in an elementary physics course to a group of freshmen in 1966 stated he had forgotten classical mechanics and it was only possible for him to teach quantum mechanics. This made perfect sense as quantum mechanics cannot be derived from classical mechanics. After introducing the Schrödinger equation a student named Peter Landesmann asked "What would be the result if mass in the Schrödinger would be a complex number?" Attempts to answer this question has unduly complicated physics. Then he introduced an additional problem with quantum mechanics in the question about how primitive longitudinal fields in physics are poorly understood.

Recently we are grateful to Glenn Westphall for his discussion on the Copenhagen view of quantum mechanics and Doug Higinbotham who explained the nuclear short range correlation data and the schemes used to model high energy electron scattering. Finally to Fabbio Mazzolai and Ganapati Myneni both impatient with the hydrogen-in-metals community triggered a reexamination of the effects hydrogen contamination in superconducting accelerator cavities that led to a result of how mass can be defined from studying the limits of quantum

diffusion in metals.

REFERENCES

Derbin, A., and B. group (2016), "The main results of the borexino experiment, arxiv:1605.06795v1 [hep-ex],".

Dirac, P. A. M. (1932), Proc.Roy. Soc. A **136**, 453.

Dodd, C., and W. E. Deeds (1968), J. App. Phys. **39**, 2829.

Einstein, A., and et. al. (1952), *The Principle of Relativity* (Dover Publications, NYC).

Feinberg, G. (1967), Phys. Rev. **159**, 1089.

Fermi, E. (1961), *notes on Quantum Mechanics* (University of Chicago Press, Chicago, Ill.).

Formaggio, J., and G. Zeller (2012), Rev. Mod. Phys. **84**, 1307, arXiv: 1305.7513v1.

Gutzwiller, M. (1990), *Chaos in Classical and Quantum Mechanics* (Springer-Verlag, NYC).

Heisenberg, W. (1930), Zeitschrift f. Physik **65**, 4.

Kac, M. (1947), American Mathematical Monthly **54** (7), 369.

Kac, M. (1949), American Mathematical Monthly **65** (1), 1.

Kac, M. (1959), *Statistical Independence in Probability, Analysis and Number Theory*, #12 The Carus Mathematical Monographs (The Math. Assoc. of America, Rahway, NJ).

Kac, M. (1966), American Mathematical Monthly **73** (4 part 2), 73.

Maxwell, J. C. (1866), *A Treatise on Electricity and Magnetism Vol II*, 3rd ed. (Dover Press, NYC) reproduction.

Miller, A. (1994), *Early Quantum Electrodynamics a source book* (Cambridge Unvi. Press, Cambridge, UK).

Pauli, W., and V. Weisskopf (1934), Helv. Phys. Acta **7**, 709.

Siegfried, R. (1983), *The Reconstruction of Electrical Conductivity Profiles using Multi frequency Eddy Current Testing*, Ph.D. thesis (Univ. of Minn., Mnpl.).

Slater, L. J. (1968), in *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, ASM 55, edited by M. Abramowitz and I. Stegun (Dept. of Commerce, Washington DC) pp. 503–536.

Srendnicki, M. (2007), *Quantum Field Theory* (Cambridge Univ. Press, Cambridge).

Wallace, J. (2009a), "Electrodynamics in iron and steel, arxiv:0901.1631v2 [physics.gen-ph],".

Wallace, J. (2009b), JOM **61** (6), 67.

Wallace, J. (2011), in *SSTIN10 AIP Conference Proceedings 1352*, edited by G. Myneni and et. al. (AIP, Melville, NY) pp. 205–312.

Wallace, J., and M. Wallace (2014), *The Principles of Matter amending quantum mechanics*, Vol. 1 (Casting Analysis Corp., Weyers Cave, VA).

Wallace, J., and M. Wallace (2015), in *Science and Technology of Ingot Niobium for Superconducting Radio Frequency Applications*, Vol. 1687, edited by G. Myneni (AIP, Melville, NY) pp. 040004–1–14, *Electrostatics*.

Wallace, J., and M. Wallace (2017), *"yes Virginia, Quantum Mechanics can be Understood"* (Casting Analysis Corp., Weyers Cave, VA).

Wallace, J., and M. J. Wallace (2011), in *SSTIN10 AIP Conference Proceedings 1352*, edited by G. Myneni and et. al. (AIP, Melville, NY) pp. 313–335.

.

History:

4 June 2016 draft 1 title *Neutrino Density Function* rejected PRL

14 June 2016 draft 2

1 July 2016 draft 3 title *Inertial Mass, Real and Complex* rejected Proc. Roy. Soc. A

27 July 2016 draft 4 title *Statistical Independence in Quantum Mechanics* rejected Rev. Mod. Phys.

1 August 2016 draft 5 posted on www.castinganalysis.com

12 September 2016 draft 6

1 August 2018 draft 7 *Refraction and its Quantum Origins* rejected J. Appl. Phys. Rev.

4 August 2019 draft 8